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Solving Reaction-Diffusion Equations with the Differential Transformation Method

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Abstract—The differential transformation technique (DTM) is used in this study to resolve reaction diffusion equations, both linear and non-linear. We have specified several instances to show how the techniques work and how reliable they are. The known forms of the series solutions allow us to get the precise answers, and we may also obtain the approximate solution by calculating it as a series with readily computable terms. This approach may alleviate computing burdens and is applicable to a wide range of linear and non-linear problems. A comparison is made between the DTM results, the VIM results, and the MATLAB solutions.

Keywords— reaction diffusion equations, differential transformation method, MATLAB

I. INTRODUCTION

In recent years there has been much interest in of systems reaction diffusion equations which occur in many applications for example mathematical biology including models for multi-spaces chemical reactions and predator prey systems. The reaction diffusion-equation and its variants have been the subject of study in various branches of physical, chemical and biological sciences. Besides its study in one dimension and higher dimensions, this all depends on the nature of the phenomenon under study as to what form of this equation (whether linear or nonlinear version) needs to be used. While a variety of simplified versions of nonlinear reaction-diffusion equations are studied [7-10, 12] in the literature, a reaction-diffusion equation with cubic nonlinearity has been found recently in several interesting applications.

Reaction-diffusion equation with nonlocal boundary conditions has been given considerable attention in recent years, and various methods have been developed for the treatment of these equations. The numerical solutions of the problem can be found in many articles. Most of the discussions in the current literatures are developed to the some type nonlocal boundary conditions problem, and much less is given to the problem with nonlocal Robin type boundary conditions. The purpose of this article is to give a numerical treatment to a class of reaction-diffusion equations with

nonlocal boundary conditions by differential transformation method.

The differential transformation is a numerical method for solving differential equations. The concept of differential transform was introduced by Zhou (1986), who was the first one to use differential transform method (DTM) in engineering applications. He employed DTM in solution of initial boundary value problems in electric circuit analysis. In recent years, concept of DTM has broadened to the problems involving partial differential equations and systems of differential equations [1-5].

Ayaz [3-5] developed this method for PDEs and obtained closed form series solutions for linear and non-linear initial value problems. The differential transforms method an analytical solution in the form of a polynomial. It is different from the traditional high order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally taken long time for large orders. The present method reduces the size of computational domain and applicable to many problems easily. A distinctive practical feature of the differential transformation method DTM is having ability to solve linear or non-linear differential equations.

Different applications for the differential transformation in the differential equation were shown by Hassan [1], where he was used the differential transformation technique which is applied to solve Eigen value problems and partial differential equations (P.D.E.) is proposed in this study. They were first, using the one-dimensional differential transformation to construct the Eigen values and the normalized Eigen functions for the differential equation of the second, and the fourth-order, and the second, using the two-dimensional differential transformation to solve P.D.E. of the first- and second-order with constant coefficients. In both cases, a set of difference equations is derived and the results were compared with the results obtained by other analytical methods.

Kumar et al (2009) discussed about the exact and numerical solutions of non-linear reaction diffusion equation by using the Cole–Hopf transformation. In this work the main focus is to get a constructive method for obtaining exact

solutions of certain classes of non-linear equations arising in mathematical biology. The method is based on the Cole–Hopf transformation of non-linear partial differential equation. With the help of this method, new exact solutions were obtained for non-linear reaction-diffusion equations of various forms, which are the generalizations of the Fisher and Burgers equations. Finally the governing partial differential equations are then solved using MATLAB. Again Kumar et al (2010) worked on the solution of reaction–diffusion equations by using homotopy perturbation method.

Application of He's homotopy perturbation method for solving the Cauchy reaction diffusion problem have also discussed by Yildirim (2009), where he presented the solution of Cauchy reaction–diffusion problem by the homotopy perturbation method.

Our work, in this paper, relies mainly on the most recently methods, DTM. The DTM method, which accurately compute the solutions in a series form or in an exact form, are of great interest to applied sciences. The effectiveness and the usefulness of method is demonstrated by finding exact solutions to the below problems 1-3, that will be investigated. However, the method has its own characteristics and significance that will be examined.

II. MATHEMATICAL FORMULATION

In this paper, consider the following one-dimensional time

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} + r u \quad (x, t) \in \Omega \subset R^2 \quad (1)$$

$$u(x, 0) = g(x), \quad x \in R \quad (2)$$

$$(0, t) = f(t), \quad \left(\frac{\partial u}{\partial x} \right)_{(1)} = f_1(t), \quad t \in R \quad (3)$$

where $u = u(x, t)$ is the concentration, $r = r(x, t)$ is the reaction parameter and $D > 0$ is the diffusion coefficient, subject to the initial or boundary conditions

The equations (1) and (2) is called the characteristic Cauchy reaction-diffusion equation in the domain $R \times R_+$, while the equations (1) and (3) is called the non-characteristic Cauchy reaction-diffusion equation in the domain $R_+ \times R$. If $r = 1$ then the equation (1) is called Kolmogorov–Petrovsky–Piskunov equations.

The DTM introduces a promising approach for many applications in various domains of science. However, DTM has some drawbacks. By using the DTM, we obtain a series solution, or we can say a truncated series solution. This series solution does not exhibit the real behaviours of the problem but gives a good approximation to the true solution in a very small region. While homotopy perturbation method can be used successfully finding the solution of linear and non-linear boundary value problems, and the system of differential equations. It may be concluded that this technique is very powerful and efficient in finding the analytical solutions for a large class of integral and differential equations. This technique provides more realistic series solutions as compared with the Adomian decomposition and variational iteration method and others techniques.

III. BASIC IDEA OF DIFFERENTIAL TRANSFORMATION METHOD

The basic definitions of the differential transformation are introduced as follows:

(i) One-dimensional differential transformation:

The differential transformation of the k^{th} derivative of a function $u(x)$ is defined as follows:

$$U(k) = \frac{1}{k!} \left[\frac{d^k u(x)}{dx^k} \right]_{x=x_0} \quad (4)$$

$u(x)$ is the original function while $U(k)$ is the transformed function.

As discussed by [1] and [23] the differential inverse transformation of $U(k)$ may defined as follows:

$$\begin{aligned} u(x) &= \sum_{k=0}^{\infty} U(k)(x - x_0)^k \end{aligned}$$

In fact, from (4) and (5), we may obtain:

$$\begin{aligned} u(x) &= \sum_{k=0}^{\infty} \frac{1}{k!} (x - x_0)^k \left[\frac{d^k u(x)}{dx^k} \right]_{x=x_0} \end{aligned} \quad (6)$$

Equation (6) implies that the concept of differential transformation is derived from the Taylor series expansion at $x = x_0$.

(ii) Two-dimensional differential transformation:

Similarly, consider a function of two variables (x, y) analytic in the domain R and let $(x, y) = (x_0, y_0)$ in this domain. The function (x, y) is then represented by one power series whose center is located at (x_0, y_0) . Hence the differential transformation of function (x, y) is having the following form:

$$U(k, h) = \frac{1}{k!h!} \left[\frac{\partial^{k+h} u(x, y)}{\partial x^k \partial y^h} \right]_{(x_0, y_0)} \quad (7)$$

where $u(x, y)$ is the original function and $U(k, h)$ is the transformed function.

The transformation is called T-function while the lower case and upper case letters represent the original and transformed functions respectively.

The differential inverse transform of (k, h) is defined as:

$$(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} U(k, h) (x - x_0)^k (y - y_0)^h \quad (8)$$

and from equation (7) and (8) it may be concluded that,

$$u(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \frac{1}{k!h!} \left[\frac{\partial^{k+h} u(x, y)}{\partial x^k \partial y^h} \right]_{(x_0, y_0)} (x - x_0)^k (y - y_0)^h \quad (9)$$

When we apply equation (8) at $(x_0, y_0) = (0, 0)$, then (8) can be written as:

$$\begin{aligned} (x, y)_{\infty}^{\infty} &= \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \frac{1}{k! h!} U(k, h) x^k y^h \quad (10) \\ u(x, y)_{\infty}^{\infty} &= \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \frac{1}{k! h!} \left[\frac{\partial^{k+h} u(x)}{\partial x^k \partial y^h} \right]_{(0,0)} x^k y^h \quad (11) \end{aligned}$$

The fundamental mathematical operations performed by two-dimensional differential transform method and that can be readily obtained through Table 1.

Table 1: Some operations of the two dimensional differential transformation.

Original function	Transformed function
$u(x, y) = v(x, y) \pm w(x, y)$	$U(k, h) = V(k, h) \pm W(k, h)$
$u(x, y) = \alpha v(x, y)$	$U(k, h) = \alpha V(k, h) \quad (\alpha = \text{Constant})$
$u(x, y) = \frac{\partial v(x, y)}{\partial x}$	$U(k, h) = (k+1)V(k+1, h)$
$u(x, y) = \frac{\partial v(x, y)}{\partial y}$	$U(k, h) = (h+1)V(k, h+1)$
$u(x, y) = \frac{\partial^{r+s} v(x, y)}{\partial x^r \partial y^s}$	$U(k, h) = \frac{(k+r)! (h+s)!}{k! h!} V(k+r, h+s)$
$u(x, y) = v(x, y)w(x, y)$	$U(k, h) = \sum_{r=0}^k \sum_{s=0}^h V(r, h-s) W(k-r, s)$
$u(x, y) = x^m y^n$	$U(k, h) = \delta(k-m, h-n) = \begin{cases} 1 & k=m \text{ and } h=n \\ 0 & \text{otherwise} \end{cases}$
$u(x, y) = e^{\lambda(x+y)}$	$U(k, h) = \frac{\lambda^{k+h}}{k! h!}$
$u(x, y) = x^m \sin(ax + b)$	$U(k, h) = \frac{a^h}{h!} \delta(k-m) \sin\left(\frac{h\pi}{2} + b\right)$
$u(x, y) = x^m e^{ay}$	$U(k, h) = \frac{a^h}{h!} \delta(k-m)$

The application of the differential transformation method (DTM) will be discussed, for solving problems (1)-(3). According to the (DTM) and the operations mathematics of the method, consider equation (1) after taking the differential transformation of both sides in the following form:

$$U(k, h) = \frac{1}{(h+1)} \left[\sum_{r=0}^k \sum_{s=0}^h \binom{k}{r} \binom{h}{s} U(r, h-s) U(k-r, s) \right] \quad (12)$$

and by applying the differential transformation to initial and boundary conditions equation (2) and (3) are obtained as follows:

$$\begin{aligned} (k, 0) &= G(k) \\ (0, h) &= F_0(h) \text{ and } U(1, h) = F_1(h) \end{aligned}$$

Problem 1: Consider the following the Kolmogorov-

Petrovsky-Piskunov (KPP) equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u \quad (x, t) \in \Omega \subset R \quad (15)$$

with initial and boundary conditions

$$u(x, 0) = e^{-x} + x = (x), \quad x \in R, \quad (16)$$

$$u(0, t) = 1 = f_0(t) \quad (17)$$

$$\frac{\partial u}{\partial x}(1, t) = e^{-t} - 1 = f_1(t), \quad t \in R \quad (18)$$

According to the differential transformation method (DTM)

When taking the differential transformation of (14), can be obtain,

$$(k, h+1) = \frac{1}{h+1} [(k+1)(k+2)U(k+2, h) - U(k, h)] \quad (19)$$

The related initial conditions (15) should be also transformed as follows

$$\sum_{r=0}^{\infty} (k, 0)^r = (1-x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots) + (k-1, h) \quad (20)$$

which gives

$$(k, 0) = \begin{cases} 1, & \text{if } k=0 \\ 0, & \text{if } k=1 \\ \frac{(-1)^k}{k!}, & \text{if } k=2, 3, 4, \dots \end{cases} \quad (21)$$

And the boundary condition (16) should be transformed as follows:

$$(0, h) = (k, h) = \begin{cases} 1 & \text{if } k=h=0 \\ 0, & \text{if otherwise} \end{cases} \quad (22)$$

and

$$\sum_{r=0}^{\infty} (1, h)t^r = (1-t + \frac{1}{2!}t^2 - \frac{1}{3!}t^3 + \frac{1}{4!}t^4 - \frac{1}{5!}t^5 + \dots) + (k, h) \quad (23)$$

which gives

$$(k, 0) = \begin{cases} 0, & \text{if } k=0 \\ -1, & \text{if } k=1 \\ \frac{(-1)^h}{h!}, & \text{if } h=2, 3, 4, \dots \end{cases} \quad (24)$$

For each

k, h substituting equations (20), (21) and (23) into equation (19) and by recursive method, all other of $U(k, h)$ are equal to zero, when substitute all values $U(k, h)$ into (12) and obtain the series for $u(x, t)$. Then rearrange the solution, and obtain the following closed form solution:

$$\begin{aligned} u(x, t) &= \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} U(k, h) x^k t^h \\ u(x, t) &= (1 - xt + \frac{1}{2} \lambda t^2 - \frac{1}{6} xt^3 + \frac{1}{24} xt^4 - \frac{1}{120} xt^5 + \dots \\ &\quad + \frac{1}{2} x^2 - \frac{1}{6} x^3 + \frac{1}{24} x^4 - \frac{1}{120} x^5 + \dots), \end{aligned}$$

$$(x, t) = xe^{-t} + e^{-x} \quad (25)$$

From equations (24) the approximate solution of the given problem (1) by using differential transformation method is having the same results as that obtained by the homotopy-perturbation method [12] and it clearly appears that the approximate solution remains closed form to exact solution.

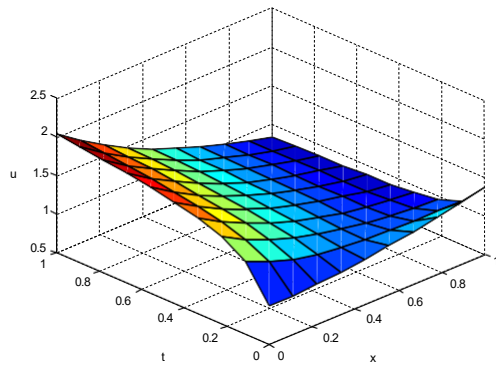


Figure 1 (a)

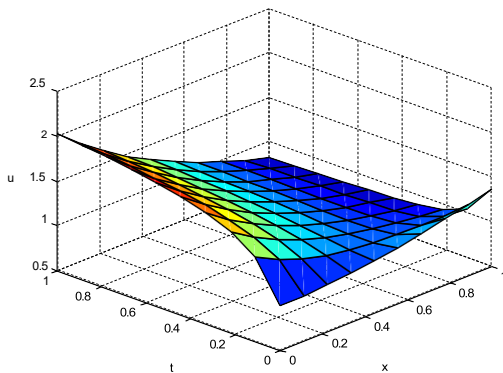


Figure 1 (b)

Figure 1(a) represents the HPM or DTM solution for the problem (1) of equation (24) from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$ while Figure 1(b) represents the by MATLAB solution of problem (1) from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$.

The accuracy of the DTM for the combined KPP equation is controllable, and absolute errors are very small with the present choice of t and x . These results also obtain by MATLAB, and both the results are compare through the figure 1(a) & (b) and there are no visible differences in the two solutions.

Example2: Consider the following the reaction-diffusion equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2tu \quad (x, t) \in \Omega \subset R$$

(26)

with initial and boundary conditions

$$u(x, 0) = e^x = (x), \quad x \in R,$$

(27)

$$u(0, t) = e^{t+t^2} = f_0(t), \quad \frac{\partial u}{\partial x}(1, t) = e^{t+t^2} = f_1(t), \quad t \in R \quad (28)$$

According to the DTM, equation (25) may be written as:

$$(k+1)(k+2)(k+2, h) = \frac{1}{h+1} \left[- \sum_{r=0}^k \sum_{s=0}^h \delta_{r, h-s-1} U(k-r, s) \right] \quad (29)$$

From the initial condition (26) can be written as

$$(k, 0) = \begin{cases} 1, & \text{if } k = 0 \\ 0, & \text{if } k = 1, 2, 3, \dots \end{cases} \quad (30)$$

And the boundary condition (27), can calculated from the following Table 2.

Table2.

$$\begin{aligned} (0, h) &= (1, h), \quad h = 0, 1, 2, 3, \dots \\ (0, 0) &= U(1, 0) = 1, \quad U(0, 1) = U(1, 1) = 1, \\ (0, 2) &= U(1, 2) = \frac{3}{2}, \quad (0, 3) = U(1, 3) = \frac{7}{6}, \quad (0, 4) = \frac{25}{24} \\ U(0, 5) &= U(1, 5) = \frac{27}{40} \\ U(0, 6) &= U(1, 6) = \frac{331}{720}, \quad U(0, 2) = U(1, 2) = \frac{1303}{5040} \dots \end{aligned}$$

Using the values of table 1 into (28), and by recursive method, we have

$$\begin{aligned} (2, 2) &= \frac{3}{4}, \quad U(3, 2) = \frac{1}{4}, \quad U(4, 2) = \frac{1}{16} \\ (5, 2) &= \frac{1}{180}, \quad U(6, 2) = \frac{1}{180}, \dots \\ (2, 3) &= \frac{1}{12}, \quad U(3, 3) = \frac{1}{36}, \quad U(4, 3) = \frac{7}{144} \\ (2, 4) &= \frac{25}{12}, \quad U(3, 4) = \frac{1}{44}, \dots \\ U(2, 5) &= \frac{49}{80}, \quad U(3, 5) = \frac{1}{80}, \quad U(4, 5) = \frac{9}{32} \dots \end{aligned}$$

Substituting all values (k, h) into (10) and obtain series for (x, t) . Then rearrange the solution, and get the following closed form solution:

$$u(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} U(k, h) x^k t^h = e^{(x+t+t^2)} \quad (31)$$

From equations (30) the approximate solution of the given problem (2) by using differential transformation method is the same results as that obtained by the homotopy-perturbation method and it clearly appears that the approximate solution remains closed form to exact solution.

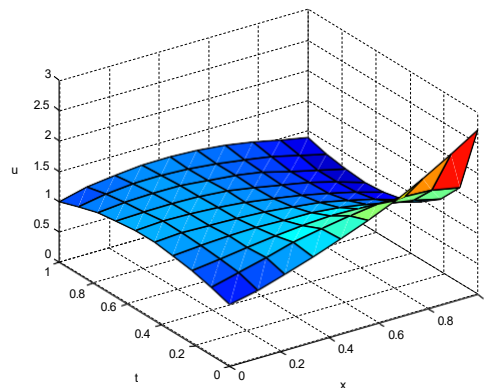


Figure 2 (a)

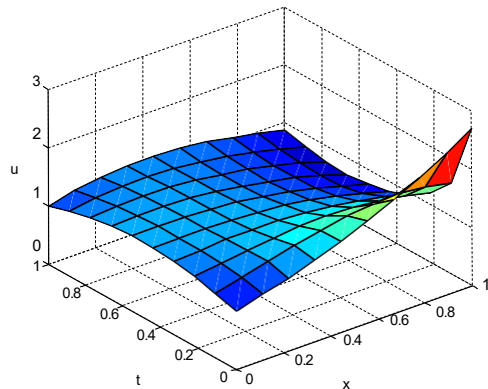


Figure 2 (b)

Figure 2(a) represents the HPM solution for the problem (1) of equation (30) from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$ while the figure 2(b) represents the MATLAB solution of Problem (2) from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$

The accuracy of the DTM for the combined KPP equation is controllable, and absolute errors are very small with the present choice of t and x . These results also obtain by MATLAB, and both the results are comparing through the figures 2(a) and (b). There are no visible differences in the two solutions.

Example3: Consider the following the reaction-diffusion:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - (4x^2 - 2t + 2) \quad (x, t) \in \Omega \subset R, \quad (32)$$

with initial and boundary conditions

$$u(x, 0) = e^{x^2} = (x), \quad x \in R, \quad (33)$$

$$u(0, t) = e^{t^2} = f_0(t) \quad (34)$$

$$\frac{\partial u}{\partial x}(1, t) = 0 = f_1(t), \quad t \in R \quad (35)$$

According to the DTM, Taking the differential transformation of (31),

$$(k, h+1) = \frac{1}{h+1} \left[\begin{aligned} &(k+1)(k+2)(k+2, h) \\ &+ 2 \sum_{r=0}^k \sum_{s=0}^h (r, h-s-1)(k-r, s) \\ &- 4 \sum_{r=0}^k \sum_{s=0}^h (r-2, h-s)(k-r, s) - 2U(k, h) \end{aligned} \right] \quad (36)$$

From the initial condition (32) of the form we may have:

$$(k, 0) = \begin{cases} 0 & \text{if } k = 1, 3, 5, \dots \\ \frac{1}{(k/2)!} & \text{if } k = 0, 2, 4, 6, \dots \end{cases} \quad (37)$$

$$(0, h) = \begin{cases} 0 & \text{if } h = 1, 3, 5, \dots \\ \frac{1}{(h/2)!} & \text{if } h = 0, 2, 4, 6, \dots \end{cases} \quad (38)$$

$$\text{and } U(1, h) = 0 \text{ for all } h \geq 0. \quad (39)$$

Using (36)-(38) into (35) by recursive method, we have the following values in Table 3 when $h \geq 1$ we have,

$$(k, h) = 0,$$

Table 3

$(2, 2) = \frac{1}{1}$	$(4, 2) = \frac{1}{2}$	$U(6, 2) = \frac{1}{6}$
$(8, 2) = \frac{1}{24}$	$U(10, 2) = \frac{1}{120}$	$\dots$
$(2, 4) = \frac{1}{24}$	$(4, 4) = \frac{1}{120}$	$U(6, 4) = \frac{1}{12}$
$(8, 4) = \frac{1}{48}$	$U(10, 4) = \frac{1}{240}$	$\dots$
$(2, 6) = \frac{1}{6}$	$(4, 6) = \frac{1}{12}$	$U(6, 6) = \frac{1}{36}$
$(8, 6) = \frac{1}{144}$	$U(10, 6) = \frac{1}{720}$	$\dots$
$(2, 8) = \frac{1}{24}$	$U(4, 8) = \frac{1}{48}$	$U(6, 8) = \frac{1}{144}$
$(8, 8) = \frac{1}{576}$	$U(10, 8) = \frac{1}{2880}$	$\dots$
$(2, 10) = \frac{1}{120}$	$U(4, 10) = \frac{1}{240}$	$U(6, 10) = \frac{1}{720}$
$(8, 10) = \frac{1}{2880}$	$(10, 10) = \frac{1}{14400}$	$\dots$

Substituting all values (k, h) into (10) and obtain series for (x, t) . Then rearrange the solution, and get the following closed form solution:

$$u(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} U(k, h) x^k t^h \quad (40)$$

From equations (39) the approximate solution of the given problem (3) by using differential transformation method is the same results as that obtained by the homotopy-perturbation method and it clearly appears that the approximate solution remains closed form to exact solution.

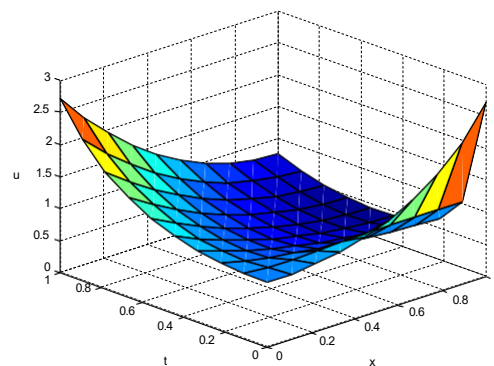


Figure 3 (a)

Figure 3(a) represents the DTM solution for the problem (3) of the equations (39), from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$

while the figure 3(b) represents the MATLAB solution of the problem (3), from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$

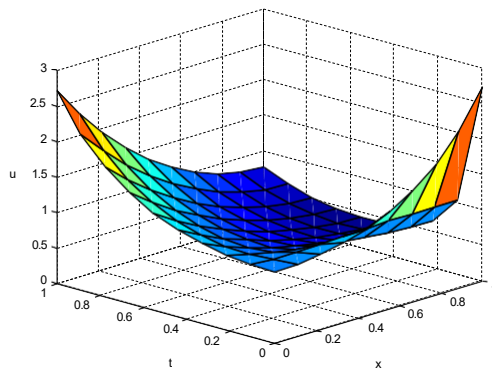


Figure 3 (b)

The accuracy of the DTM for the combined KPP equation is controllable, and absolute errors are very small with the present choice of t and x . These results also obtain by MATLAB, and both the results are comparing through the figures 3(a) and (b). There are no visible differences in the two solutions.

IV. CONCLUSIONS

Reaction–diffusion equations have special importance in engineering and sciences and constitute a good model for many systems in various fields. In this work, we studied differential transformation method for solving reaction–diffusion equations. This method is applied to solve three boundary value problems. All problems, we obtained closed form exact series solutions. The differential transformation method is so powerful and efficient that they both give approximations of higher accuracy and closed form solutions if existing. It is observed that the method is an effective and reliable tool for the solution of such problems.

Further the results of our example tell us the successfully method can be alternative way for the solution of the linear and non-linear higher-order initial and boundary value problems. In fact, DTM is very efficient methods to find the numerical and analytic solutions of differential-difference equations, delay differential equations as well as integral equations. The results show that DTM is a powerful mathematical tool for solving linear and nonlinear partial differential equations, and therefore, can be widely applied in engineering problems. In work MATLAB is used to calculate the series obtained from the DTM.

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