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Research Paper

THERMO DIFFUSION EFFECT ON HEAT AND MASS TRANSFER COUETTE FLOW THROUGH POROUS MEDIUM WITH FINE DUST

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Abstract

This paper deals with the effects of Soret and dusty fluid on a Couette flow confined between two porous plates. The fluid taken is a viscous, incompressible dust laden. The dust particles are assumed to be very fine and hence chemically inert. The flow model is described by two phase equations comprising equations of momentum, diffusion equation and energy equation. The governing partial differential equations are converted into dimensionless equations. The impact of various physical parameters are solved analytically to determine all the important flow characteristics.

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1 Introduction

Heat and mass transfer is important in many industrial and technological processes. In manufacturing and metallurgical processes, heat and mass transfer occur simultaneously. Kohler[1].This is among the very first and widely referenced papers that properly defines the heat and mass transfer problem in a binary liquid mixture in a Couette flow system, with specific inclusion of the Soret and Dufour effects. It provides the basis for much of the later analytical research.[2] Yahata,This paper presents a complete analytical solution of the velocity, temperature, and concentration fields in a binary Couette fluid flow with the Soret effect. It's the first reference people usually go to for the linear, steady-state analysis of the problem. [3] Hurle was more interested in convection, this paper is a basis for understanding the interaction between the Soret effect and buoyancy forces (double-diffusive convection). This is important when temperature gradients in a horizontal Couette flow apparatus can cause instability. Gaikwad [4]This work generalizes the classical problem to non-Newtonian (Couple stress) fluids. It examines linear as well as non-linear stability and demonstrates the effect of fluid rheology on Soret-driven heat and mass transfer. Srinivas [5] nvestigates the combined influence of Soret and Dufour (diffusion-thermo) in a vertically positioned channel, where buoyancy effects are important. This is a signature paper for use in vertical chemical reactors or material processing.[6] Awad, Addresses the stability of a Maxwell viscoelastic fluid in the presence of cross-diffusion effects. It demonstrates how fluid elasticity can manipulate the stability thresholds and heat/mass transfer rates. Mallesh[7] A contemporary example that involves several intricacies: Magnetohydrodynamics (MHD), a Casson non-Newtonian fluid (such as blood), porous media, and peristaltic pumping. This is particularly applicable to biomedical and geothermal uses. Platten[8] Although not solely pertaining to Couette flow, this review gives a very good overview of the experimental techniques and results of the Soret effect, which is necessary in the validation of the theoretical models applied to Couette flow research..[9] Gayathri et al,discussed the Three-dimensional couette Flow of dusty Fluid with heat transfer in the presence of magnetic Field. Influence of various physical parameters of flow field on velocity, temperature, concentration, coefficients of skin-friction, Nusselt number and Sherwood number have been discussed through graphs and tables. The objective of the present research work is to examine effects of Soret flow of dusty fluid along an porous plate immersed in porous medium variable temperature and variable mass diffusion. Partial differential equationsof dimensionless governing equations of flow have been solved by pertubutation method.

2. Flow Description and Governing Equations

Consider the three dimensional Couette flow of a viscous incompressible dusty fluid between two infinite parallel flat porous plates including thermo diffusion effect with heat and mass transfer. The coordinate system is introduced with its origin on the lower stationary plate lying horizontally on the $xy-$ plane and the upper plate is placed at a distance d which is subjected to a uniform motion U. The y – axis is taken perpendicular to the plates. The lower and the upper plates are assumed to be at constant temperatures T_0 and T_1 respectively, with $T_1 > T_0$. The upper plate is subjected to a constant suction V_0 and the lower to a transverse exponential injection velocity distribution of the form $v = v_0(1 + \epsilon e^{\alpha Z})$.

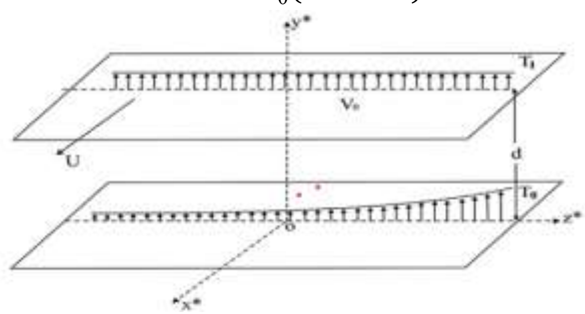

Fig 1. Schematic Diagram of the flow configuration

The basic equations that govern the steady three dimensional Couette flow of viscous fluid including thermo diffusion effect are given as:

For Fluid phase

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho[(\vec{q}^*, \nabla)\vec{q}^*] = -\nabla^* p^* + \mu \nabla^{*^2} q^- + kN(\vec{q}_p^v - \vec{q}) \tag{2}$$

$$\rho c_p(\vec{q}^* \cdot \nabla^*)T = K\nabla^{*^2} T + k_p N(T_p^* - T^*) \tag{3}$$

$$(q^* \cdot \nabla^*)C = \mathrm{D}\nabla^2 C + \frac{D_m K_T}{T_m}\nabla^2 T \tag{4}$$

For Particle phase

$$\nabla^* \cdot \vec{q}_p^* = 0 \tag{5}$$

$$m_p N(\vec{q}_p^* \cdot \nabla^*)\vec{q}_p^* = kN(\vec{q}^* - \vec{q}_p^*) \tag{6}$$

$$\rho_p C_{pp}(\vec{q}_p^* \cdot \nabla^*)T_p^* = h_p N(T^* - T_p^*) \tag{7}$$

with the corresponding boundary conditions

$$u = 0; \quad v = v_0(1 + \epsilon e^{\alpha Z}); \quad w = 0; \quad T = T_0; \quad C = C_0$$

$$u_p = 0; \quad v_p = v_0(1 + \epsilon e^{\alpha Z}); \quad w_p = 0; T_p = T_0 \qquad \text{at } y = 0$$

$$u = u_0; \quad v = v_0; \quad w = 0; \quad T = T_1; \quad C = C_1$$

$$u_p = u_0; \quad v_p = v_0; \quad w_p = 0; T_p = T_1 \qquad \text{at } y = 1 \tag{8}$$

Here, p is the non-dimensional pressure,
ρ is the density of the fluid,
N is the number density of the dust particle,
k is the stokes drag constant,
D is the mass diffusion coefficient,
D_m is the mass diffusivity rate
K is the thermal conductivity of the fluid,
K_T is the thermo diffusion ratio,
T_m is the mean fluid temperature,
C_p is the Specific heat at constant pressure,
μ is the coefficient of viscosity,
v_0 is the suction velocity,
m is the mass of the dust particle, respectively.

By introducing the following dimensionless parameters,

$$(x^*, y^*, z^*) = d(x, y, z);\ p^* = \rho_0 v_0^2 p\ ;\ (u^*, v^*, w^*) = v_0(u, v, w)$$

$$(u_d^*, v_d^*, w_d^*) = v_0(u_d, v_d, w_d);\ \frac{T - T_0}{T_1 - T_0} = \theta;\quad \frac{C - C_0}{C_1 - C_0} = \phi;\quad \frac{T_p - T_0}{T_1 - T_0} = \theta_p$$

Hence the non-dimensional equations are

For Fluid phase

$$\nabla \cdot \vec{q} = 0 \tag{9}$$

$$\langle \vec{q} \cdot \nabla \rangle \vec{q} = -\nabla \rho + \frac{1}{R_e}\nabla^2 \vec{q} + \frac{f}{\tau}(\vec{q}_p - \vec{q}) \tag{10}$$

$$(\vec{q} \cdot \nabla)\theta = \frac{1}{p_r}\frac{1}{R_e}\nabla^2\theta + \delta(\theta_p - \theta) \tag{11}$$

$$(\vec{q} \cdot \nabla)\phi = \frac{1}{S_c}\frac{1}{R_e}\nabla^2\phi + Sr\nabla^2\theta \tag{12}$$

For Dust phase

$$\nabla \cdot \vec{q}_p = 0 \tag{13}$$

$$(\vec{q}_p \cdot \nabla)\vec{q}_p = \frac{1}{\tau}(\vec{q} - \vec{q}_p) \tag{14}$$

$$(\vec{q}_p \cdot \nabla)\theta_p = \delta\lambda\beta(\theta - \theta_p) \tag{15}$$

Boundary Conditions are

$$u = 0;\quad v = 1 + \epsilon e^{\alpha z};\quad w = 0;\quad \theta = 0;\quad \phi = 0$$

$$u_p = 0;\quad v_p = 1 + \epsilon e^{\alpha z};\quad w_p = 0;\quad \theta_p = 0 \qquad \text{at } y = 0$$

$$u = 1;\quad v = 1;\quad w = 0;\quad \theta = 1;\quad \phi = 1$$

$$u_p = 1;\quad v_p = 1;\quad w_p = 0;\quad \theta_p = 1 \qquad \text{at } y = 1 \tag{16}$$

The equations (9)-(15) can be written in component form as

For Fluid phase

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{17}$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{R_e}\left[\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + \frac{f}{\tau}(u_p - u) \tag{18}$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{R_e}\left[\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] + \frac{f}{\tau}(v_p - v).=$$

$$-\frac{\partial p}{\partial z}+\frac{1}{R_e}\left[\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right]+\frac{f}{\tau}\left(w_p-w\right) \quad (19)$$

$$v\frac{\partial\theta}{\partial y}+\omega\frac{\partial\theta}{\partial z}=\frac{1}{P_rR_e}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)+\delta\left(\theta_p-\theta\right) \quad (20)$$

$$v\frac{\partial\phi}{\partial y}+w\frac{\partial\phi}{\partial z}=\frac{1}{S_c\mathrm{Re}}\left[\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right]+Sr\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right) \quad (21)$$

For Particle phase

$$\frac{\partial V_p}{\partial y}+\frac{\partial\omega_p}{\partial z}=0 \quad (22)$$

$$\left(v_p\frac{\partial u_p}{\partial y}+w_p\frac{\partial u_p}{\partial z}\right)=\frac{1}{\tau}\left(u-u_p\right) \quad (23)$$

$$\left(v_p\frac{\partial v_p}{\partial y}+w_p\frac{\partial v_p}{\partial z}\right)=\frac{1}{\tau}\left(v-v_p\right) \quad (24)$$

$$\left(v_p\frac{\partial w_p}{\partial y}+w_p\frac{\partial w_p}{\partial z}\right)=\frac{1}{\tau}\left(\mathrm{w}-w_p\right) \quad (25)$$

$$\left(v_\rho\frac{\partial\theta_\rho}{\partial y}+\omega_p\frac{\partial\theta_\rho}{\partial z}\right)=\delta\lambda\beta\left(\theta-\theta_\rho\right) \quad (26)$$

3. Solution of the problem

In order to solve the differential equation (17) to (28), we assume the solution in the following form

$$u(y,z,t) = u_0(y)+\epsilon u_1(y,z,t)+\epsilon^2u_2(y,z,t)+\cdots$$
$$v(y,z,t) = v_0(y)+\epsilon v_1(y,z,t)+\epsilon^2v_2(y,z,t)+\cdots$$
$$w(y,z,t)=w_0(y)+\epsilon w_1(y,z,t)+\epsilon^2w_2(y,z,t)+\cdots$$
$$u_p(y,z,t)=u_{p_0}(y)+\epsilon u_{p1}(y,z,t)+\epsilon^2u_{p2}(y,z,t)+\cdots$$
$$v_p(y,z,t) = v_{p_0}(y)+\epsilon v_{p_1}(y,z,t)+\epsilon^2v_{p_2}(y,z,t)+\cdots$$
$$w_p(y,z,t)=w_{p_0}(y)+\epsilon w_{p_1}(y,z,t)+\epsilon^2w_{p_2}(y,z,t)+\cdots$$
$$p(y,z,t)=p_0(y)+\epsilon p_1(y,z,t)+\epsilon^2p_2(y,z,t)+ \quad (27)$$

When $\epsilon = 0$, the problem reduces to

$$\frac{\partial v_0}{\partial y}=0,\Rightarrow v_0=1 \quad (28)$$

$$\frac{\partial^2u_0}{\partial y^2}-Re\frac{\partial u_0}{\partial y}-\frac{f}{\tau}Reu_0(y)=-\frac{f}{\tau}Re\ u_{p_0}(y) \quad (29)$$

$$\frac{\partial P_0}{\partial y}=0\Rightarrow\ p_0=k=1 \quad (30)$$

$$\frac{\partial^2\theta_0}{\partial y^2}-PrRe\frac{\partial\theta_0}{\partial y}-PrRe\ \delta\theta_0(y)=-PrRe\ \delta\ \theta_{p_0} \quad (31)$$

$$\frac{\partial\varphi_0}{\partial y}=\frac{1}{Sc\ Re}\frac{\partial^2\varphi_0}{\partial y^2}+Sr\frac{\partial^2\theta_0}{\partial y^2} \quad (32)$$

$$\frac{\partial v_{p_0}}{\partial y}=0\ \Rightarrow v_{P_0}=1 \quad (33)$$

$$\frac{\partial u_{p_0}}{\partial y}=\frac{1}{\tau}\left(u_0(y)-u_{p_0}(y)\right) \quad (34)$$

$$\frac{\partial\theta_{p_0}}{\partial y}=\delta\,\lambda\,\beta\theta_0(y)-\delta\,\lambda\,\beta\theta_{p_0} \quad (35)$$

$$\frac{\partial^2\varphi_0}{\partial y^2}-Sc\ Re\ \frac{\partial\varphi_0}{\partial y}=0 \quad (36)$$

Hence the solution is given by

$$u_0(y)=A_3+A_4e^{m_1y}+A_5e^{m_2y} \quad (37)$$

$$u_{p_0}=A_6e^{\frac{-y}{\tau}}+A_3+A_4\frac{e^{m_1y}}{(m_1\tau+1)}+A_5\frac{e^{m_2y}}{(m_2\tau+1)} \quad (38)$$

$$\theta_0(y) = A_7 + A_8 e^{m_3 y} + A_9 e^{m_4 y} \quad (39)$$

$$\theta_{p_0} = A_{10} e^{-\lambda_\beta \delta y} + A_7 + A_8 B_{17} e^{m_3 y} + A_9 B_{18} e^{m_4 y} \quad (40)$$

When $\epsilon \neq 0$, substituting (27) in equations (1) –(7) comparing the coefficients of ϵ and neglecting the higher order terms, we get the following equations

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad (41)$$

$$\frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_0}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) + \frac{f}{\tau}\left(u_{p_1} - u_1\right) \quad (42)$$

$$\frac{\partial v_1}{\partial y}\mathrm{Re} = -\mathrm{Re}\frac{\partial p_1}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) + \frac{f}{\tau}\mathrm{Re}\left(v_{p_1} - v_1\right) \quad (43)$$

$$\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) + \frac{f}{\tau}\left(w_{p_1} - w_1\right) \quad (44)$$

$$\frac{\partial \theta_1}{\partial y} + v_1 \frac{\partial \theta_0}{\partial y} = \frac{1}{\mathrm{PrRe}}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) + \delta\left(\theta_{p_1} - \theta_1\right) \quad (45)$$

$$\frac{\partial \phi_1}{\partial y} + v_1 \frac{\partial \phi_0}{\partial y} = \frac{1}{S_c\mathrm{Re}}\left(\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) \quad (46)$$

$$\frac{\partial v_{p_1}}{\partial y} + \frac{\partial w_{p_1}}{\partial z} = 0 \quad (47)$$

$$\frac{\partial u_{p_1}}{\partial y} + v_{p_1} \frac{\partial u_{p_0}}{\partial y} = \frac{1}{\tau}\left(u_1 - u_{p_1}\right) \quad (48)$$

$$\frac{\partial v_{p_1}}{\partial y} = \frac{1}{\tau}\left(v_1 - v_{p_1}\right) \quad (49)$$

$$\frac{\partial w_{p_1}}{\partial y} = \frac{1}{\tau}\left(w_1 - w_{p_1}\right) \quad (50)$$

$$\frac{\partial \theta_{p_1}}{\partial y} + v_1 \frac{\partial \theta_{p_0}}{\partial y} = \lambda_\beta \delta\left(\theta_1 - \theta_{p_1}\right) \quad (51)$$

To solve the equations (42) to (58) we assume the following form

$$f_1(y,z) = e^{\alpha z} \cdot f_{11}(y) \quad (52)$$

$$\omega_{p_{11}} = \left(-\frac{1}{\alpha}\right)\frac{\partial v_{p_{11}}}{\partial y} \quad (53)$$

$$\omega_{11} = \left(-\frac{1}{\alpha}\right)\frac{\partial v_{11}}{\partial y} \quad (54)$$

where $f_1(y,z) = \{u_1; v_1; w_1; u_{p1}; v_{p1}; w_{p1}; \theta_1; \theta_{p_1};\ \phi_1\}$ and

$$f_{11}(y) = \{u_{11}; v_{11}; w_{11}; u_{p11}; v_{p11}; w_{p11}; \theta_{11}; \theta_{p_{11}};\ \phi_{11}\}$$

Corresponding boundary conditions are

$$u_{11} = 0;\quad v_{11} = 1;\quad w_{11} = 0;\quad \theta_{11} = 0;\quad \phi_{11} = 0$$

$$u_{p_{11}} = 0;\ v_{p_{11}} = 1;\ w_{p_{11}} = 0;\ \theta_{p_{11}} = 0 \qquad at\ y = 0$$

$$u_{11} = 0;\quad v_{11} = 0;\quad w_{11} = 0;\quad \theta_{11} = 0;\quad \phi_{11} = 0$$

$$u_{p_{11}} = 0;\ v_{p_{11}} = 0;\ w_{p_{11}} = 0;\ \theta_{p_{11}} = 0 \qquad at\ y = 1 \quad (55)$$

using the boundary conditions equations (55) and also using equations (52), (53) and (54) following set of equations are obtained

$$p_{11} = A_{11} e^{i\alpha y} + A_{12} e^{-i\alpha y} \quad (56)$$

$$V_{11} = A_{13} e^{m_5 y} + A_{14} e^{m_6 y} + A_{15} e^{m_7 y} + B_{31} e^{i\alpha y} \quad (57)$$

$$V_{P11} = A_{13}\left(\frac{1}{\tau m_5 + 1}\right) e^{m_5 y} + A_{14} e^{m_6 y} + A_{16} e^{-\frac{y}{\tau}}$$

$$+A_{15}\frac{1}{(\tau m_7 + 1)} e^{m_7 y} + B_{31}\frac{1}{(i\tau\alpha + 1)} e^{i\alpha y} \quad (58)$$

$$u_{11} = A_{17}e^{m_5y} + A_{18}e^{m_6y} + A_{19}e^{m_7y} + B_{56}e^{(m_1+m_5)y} + B_{57}e^{(m_1+m_6)y} + B_{58}e^{(m_1+m_7)y} + B_{59}e^{(m_2+m_5)y} + B_{60}e^{(m_2+m_6)y} + B_{61}e^{(m_2+m_7)y} + B_{62}e^{(i\alpha+m_1)y} + B_{63}e^{(i\alpha+m_2)y} + B_{64}e^{\left(m_1-\frac{1}{\tau}\right)y} + B_{65}e^{\left(m_2-\frac{1}{\tau}\right)y}. \quad (59)$$

$$u_{p11} = A_{20}e^{-\frac{y}{\tau}} + A_{17}B_{81}e^{m_5y} + A_{18}B_{82}e^{m_6y} + A_{19}B_{83}e^{m7y} + B_{84}e^{(m_1+m_5)y)} + B_{85}e^{(m_1+m_6)y} + B_{86}e^{(m_1+m_7)y} + B_{87}e^{(m_2+m_5)y} + B_{88}e^{(m_2+m_6)y} + B_{89}e^{(m_2+m_7)y} + B_{90}e^{(i\alpha+m_1)y} + B_{91}e^{(i\alpha+m_2)y} + B_{92}e^{(m_1-1/\tau)y} + B_{93}e^{(m_2-1/\tau)y} + B_{94}e^{(m_5-1/\tau)y} + B_{95}e^{(m_6-1/\tau)y} + B_{96}e^{(m_7-1/\tau)y} + B_{97}e^{-\frac{2}{\tau}y} + B_{98}e^{(i\alpha-1/\tau)y} \quad (60)$$

$$\theta_{11} = A_{21}e^{m_8y} + A_{22}e^{m_9y} + A_{23}e^{m_{10}y}B_{131}e^{(m_5+m_3)y} + B_{132}e^{(m_3+m_6)y} + B_{133}e^{(m_3+m_7)y} + B_{134}e^{(m_3+i\alpha)y} + B_{135}e^{(m_4+m_5)y} + B_{136}e^{(m_4+m_6)y} + B_{137}e^{(m_4+m_7)y} + B_{138}e^{(m_4+i\alpha)y} + B_{139}e^{-\left(\lambda\delta\beta+\frac{1}{\tau}\right)y} + B_{140}e^{\left((m_3-\frac{1}{\tau}\right)y} + B_{141}e^{\left((m_4-\frac{1}{\tau}\right)y} + B_{142}e^{((m_5-\lambda\delta\beta)y} + B_{143}e^{((m_6-\lambda\delta\beta)y} + B_{144}e^{((m_7-\lambda\delta\beta)y} + B_{145}e^{(i\alpha-\lambda\delta\beta)y} \quad (61)$$

$$\theta_{p_{11}} = A_{21}B_{146}e^{m_8y} + A_{22}B_{147}e^{m_9y} + A_{23}e^{m_{10}y}B_{148} + A_{23}e^{-\lambda\delta\beta} + B_{149}e^{(m_3+m_5)y} + B_{150}e^{(m_3+m_6)y} + B_{151}e^{(m_3+m_7)y} + B_{152}e^{(m_3+i\alpha)y} + B_{153}e^{(m_4+m_5)y} + B_{154}e^{(m_4+m_6)y} + B_{155}e^{(m_4+m_7)y} + B_{156}e^{(m_4+i\alpha)y} + B_{157}e^{-\left(\lambda\delta\beta+\frac{1}{\tau}\right)y} + B_{158}e^{\left((m_3-\frac{1}{\tau}\right)y} + B_{159}e^{\left((m_4-\frac{1}{\tau}\right)y} + B_{160}e^{((m_5-\lambda\delta\beta)y} + B_{161}e^{((m_6-\lambda\delta\beta)y} + B_{162}e^{((m_7-\lambda\delta\beta)y} + B_{163}e^{(i\alpha-\lambda\delta\beta)y} \quad (62)$$

$$\phi_1 = A_{25}e^{m_{11}y} + A_{26}e^{m_{12}y} + B_{182}e^{(m_5+ScRe)y} + B_{183}e^{(m_6+ScRe)y} + B_{184}e^{(m_7+ScRe)y} + B_{185}e^{(i\alpha+ScRe)y} + B_{186}e^{(m_5+m_3)y} + B_{187}e^{(m_3+m_6)y} + B_{187}e^{(m_3+m_7)y} + B_{189}e^{(m_3+i\alpha)y} + B_{190}e^{(m_4+m_5)y} + B_{191}e^{(m_4+m_6)y} + B_{192}e^{(m_4+m_7)y} + B_{193}e^{(m_4+i\alpha)y} \quad (63)$$

Skin Friction

The skin friction at the wall due to main flow is given by

$$\tau_x = -\left(\frac{du}{dy}\right)_{y=0} = -\left(\frac{du_0}{dy}\right)_{y=0} + \in\left(\frac{du_1}{dy}\right)_{y=0} + O(\in^2)$$

The skin friction at the wall due to cross flow is given by

$$\tau_z = -\left(\frac{du}{dy}\right)_{y=1} = -\left(\frac{du_0}{dy}\right)_{y=1} + \in\left(\frac{du_1}{dy}\right)_{y=1} + O(\in^2)$$

Nusselt number

The rate of heat transfer is

$$q_W = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

The non dimensional form of Nusselt number is

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0} = -\left(\frac{d\theta_0}{dy}\right)_{y=0} + \epsilon\left(\frac{d\theta_1}{dy}\right)_{y=0} + O(\epsilon^2)$$

4. Results and Discussion

This paper discusses the three dimensional heat and mass transfer flow of viscous incompressible fluid with fine dust considering thermo diffusion effect. The main flow velocity, temperature field and Concentration field are discussed with various pertinent parameters. These characteristics are analyzed with the help of graphs.

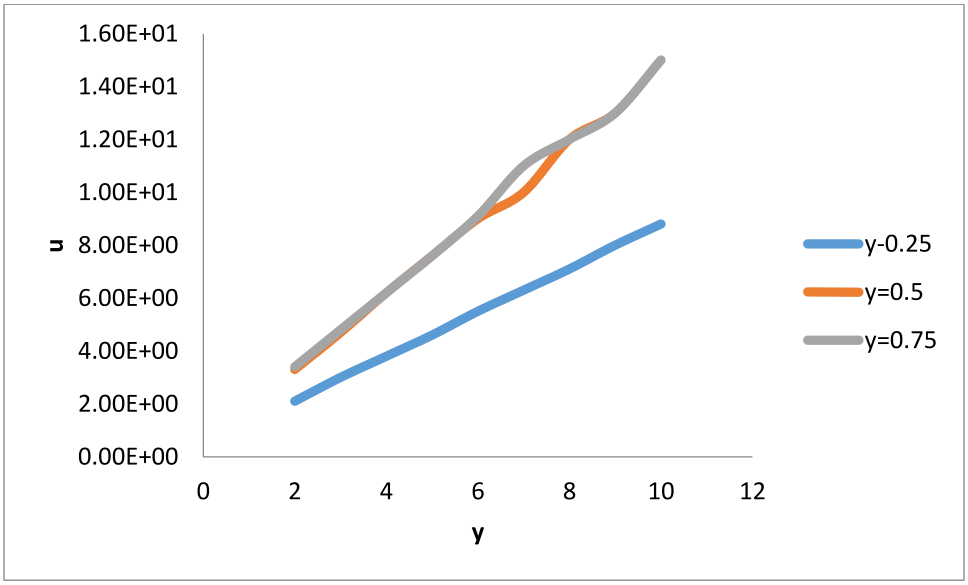

Fig 2: Main flow velocity for various y (Fluid phase)

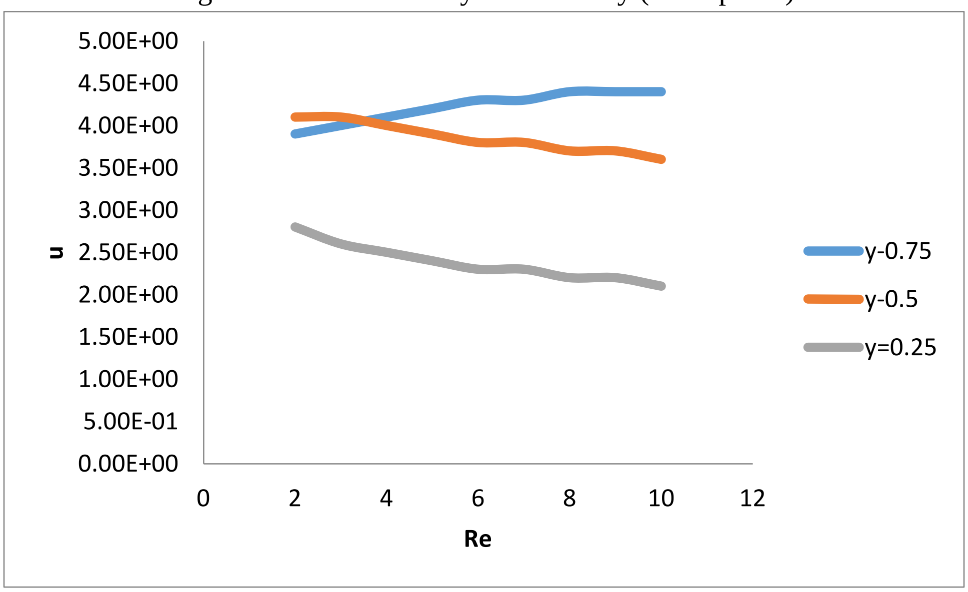

Fig 3: Main flow velocity for various y (Fluid phase)

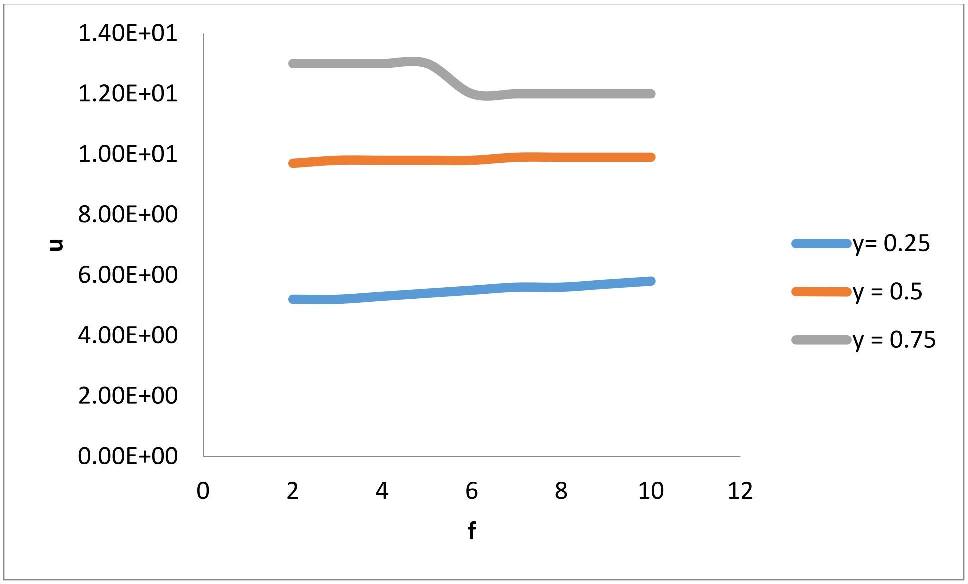

Fig 4: Main flow velocity for various y (Fluid phase)

The impact of main flow velocity on fluid phase for various Reynolds number, mass concentration parameter and relaxation time parameter are discussed through Fig 4.2 – 4.4 for various y. We can conclude that, main flow velocity at the fluid phase increases for enhancing mass concentration parameter and relaxation time parameter, depreciates with the enhancement of Reynolds number for various y.

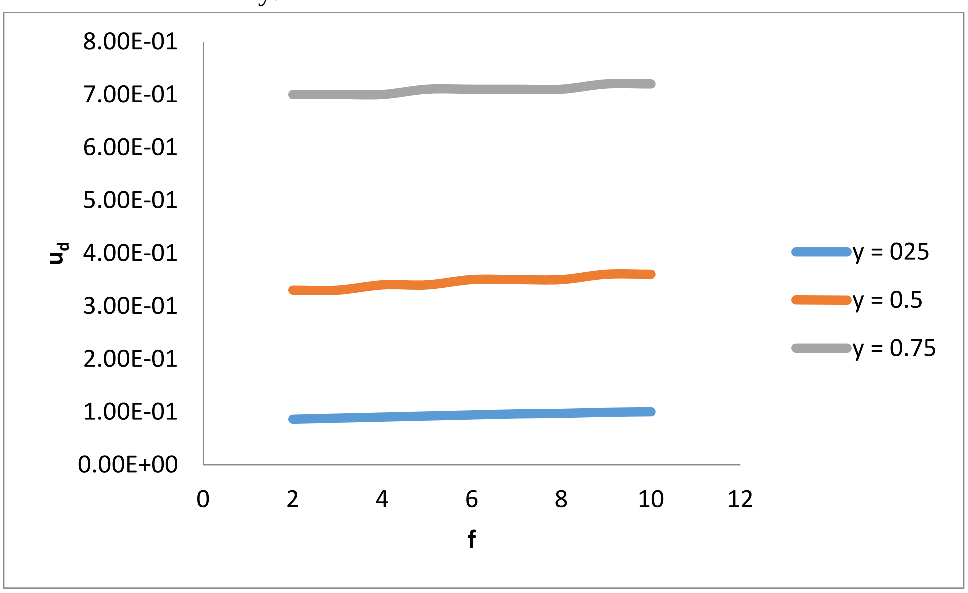

Fig 5: Main flow velocity for various y (Particle phase)

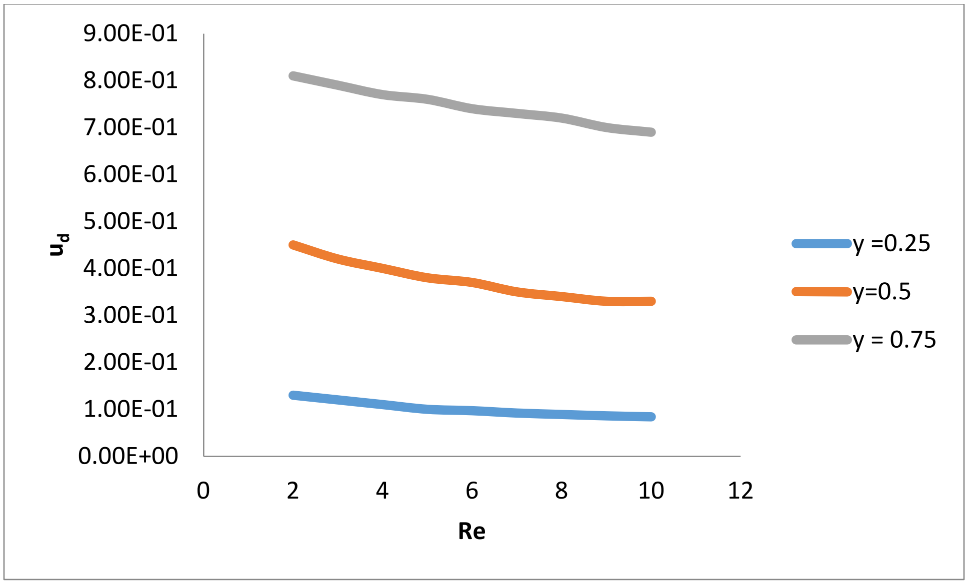

Fig 6: Main flow velocity for various y (Particle phase)

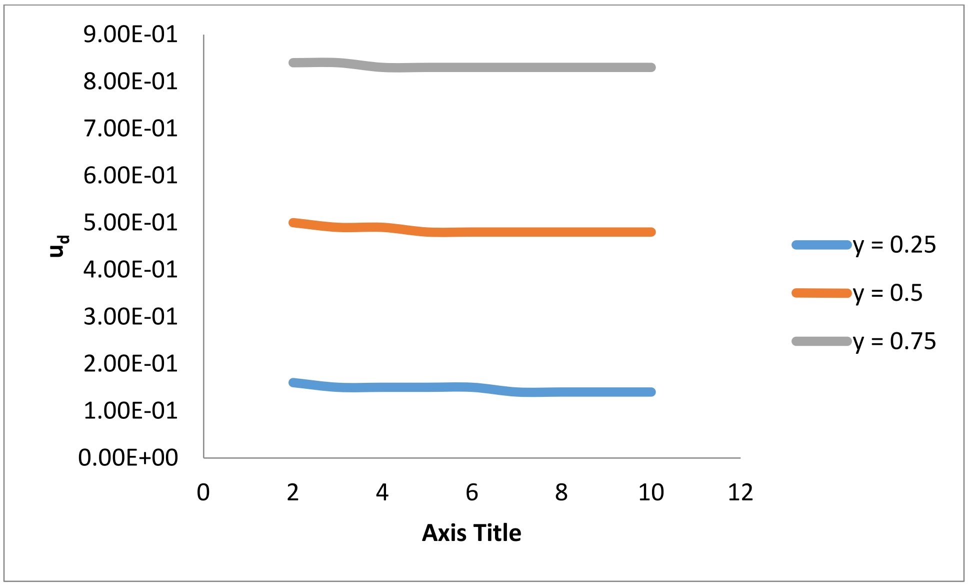

Fig 7: Main flow velocity for various y (Particle phase)

Fig 5 – 7 analyzes the effect of mass concentration parameter, Reynolds number and relaxation time parameter on main flow velocity at particle phase for various y. This is readily understood from the above figures that, the main flow velocity increases with the increase in these parameters for various y.

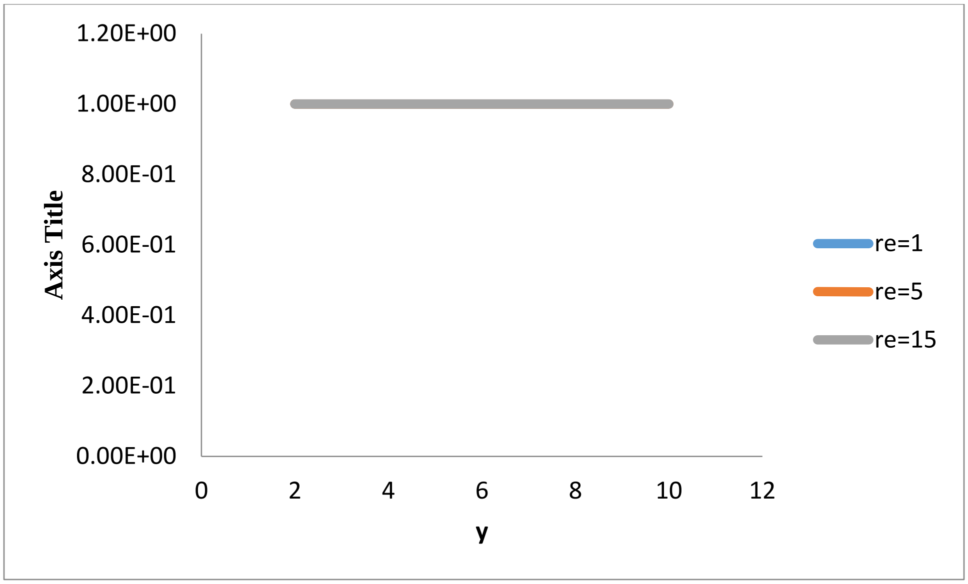

Fig 8: Temperature profile at fluid phase for various Re

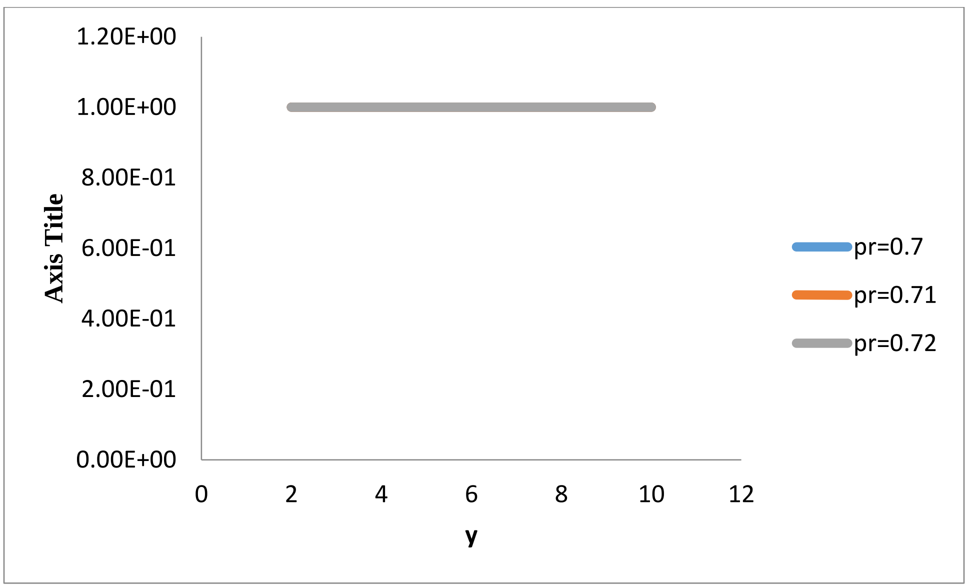

Fig 9: Temperature profile at fluid phase for various Pr

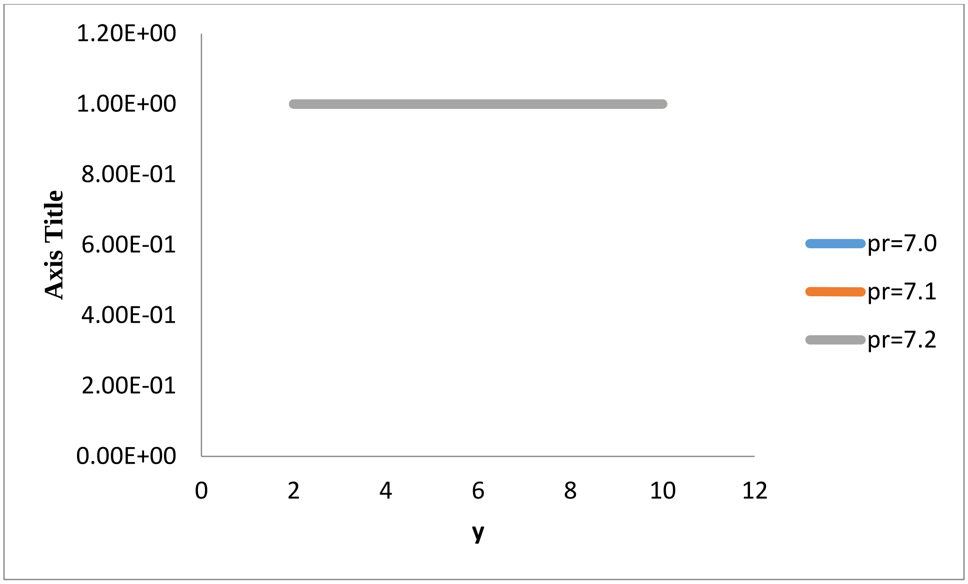

Fig 10: Temperature profile at fluid phase for various Pr (Water)

Fig 8 – 10 discusses the impact of temperature filed at fluid phase for various Reynolds number, Prandtl number (Air, Water). For increasing values of Re and Pr, it is observed that, no significant changes are found in the temperature profile.

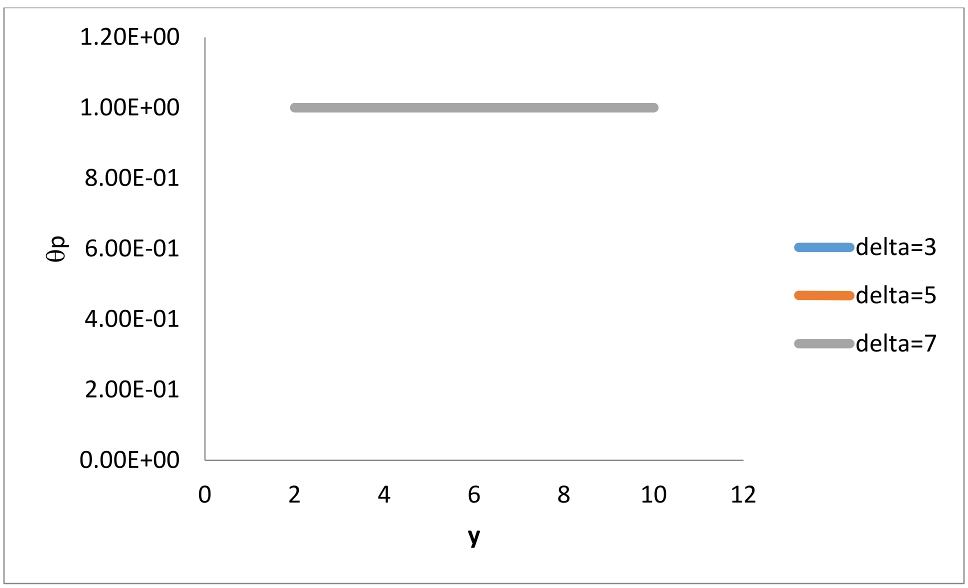

Fig 11: Temperature profile at particle phase for various δ (Re = 1)

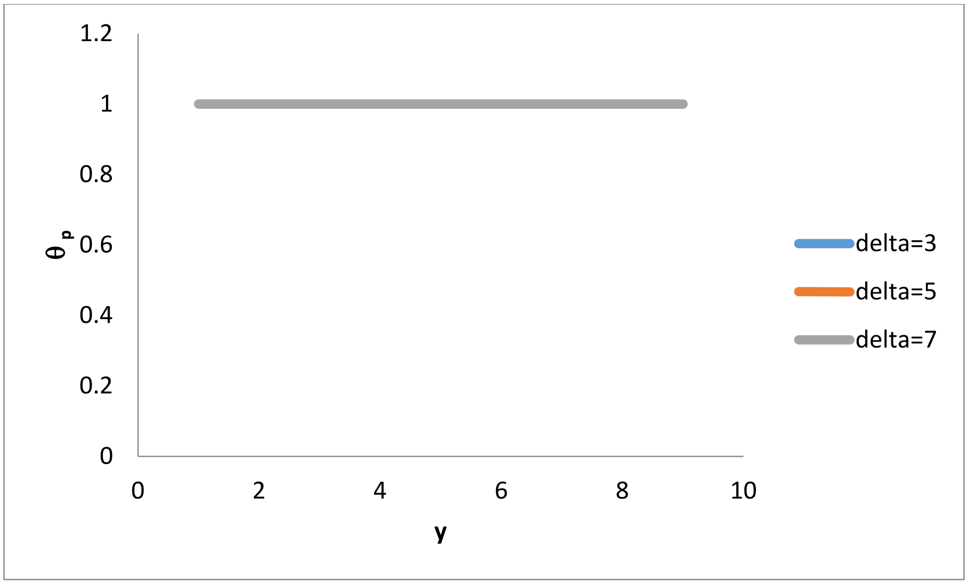

Fig 12: Temperature profile at particle phase for various δ (Air)

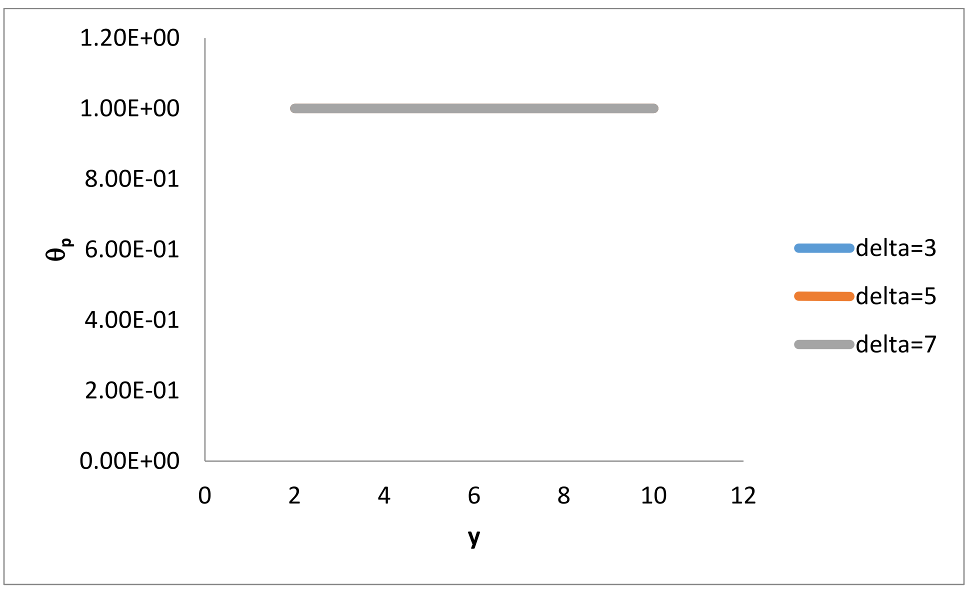

Fig 13: Temperature profile at particle phase for various δ (Water)

The temperature profile at particle phase for various δ are shown through Fig. 11 – 13. For increasing value of δ for fixed Reynolds Number (Re = 1) and Prandtl number for Air and Water, no significant changes are identified in the graphs.

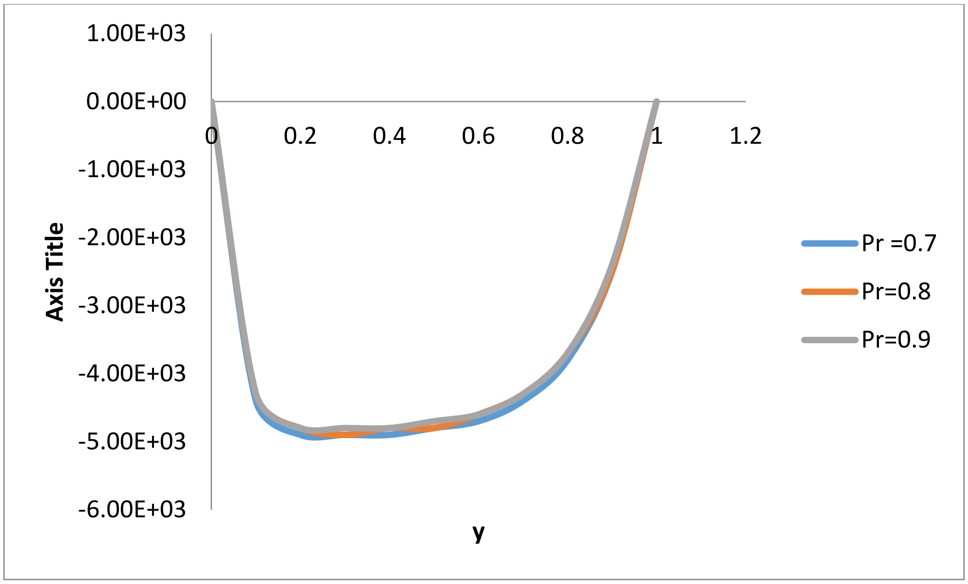

Fig 14: Concentration profile for various Pr (Air)

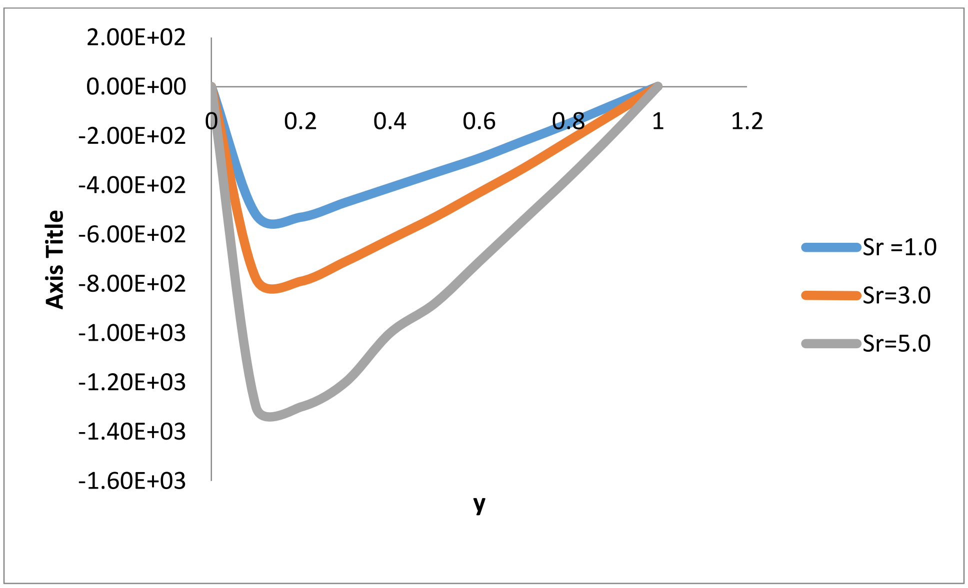

Fig 15: Concentration profile for various Sr (Air)

Fig. 14 and 15 discusses the impact of Concentration on various Prandtl number and Soret number. We can observe from these figures that, concentration is found to be decreasing with increasing Prandtl number and enhances due to the increase of Soret number.

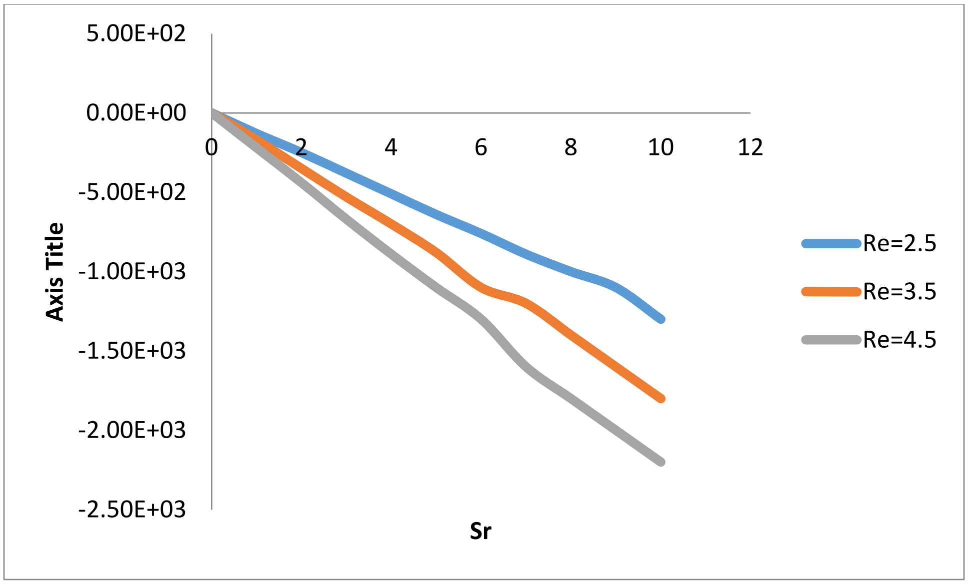

Fig 16: Concentration profile for various Re

Fig 16 discusses the impact of Concentration field on various Reynolds number. It is found that, concentration profile decreases with increasing Soret number and Reynolds number.

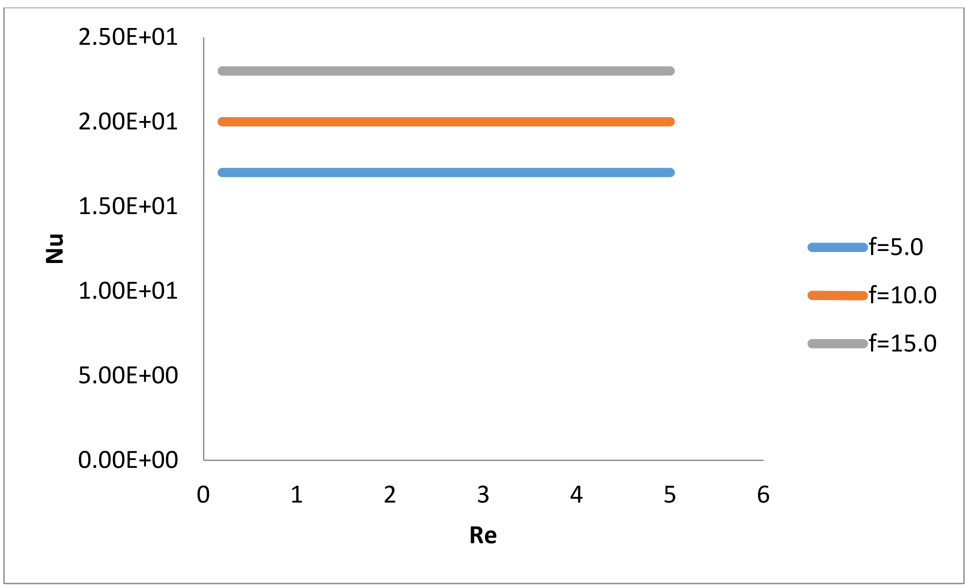

Fig 17: Nusselt number for various *f* at y = 0

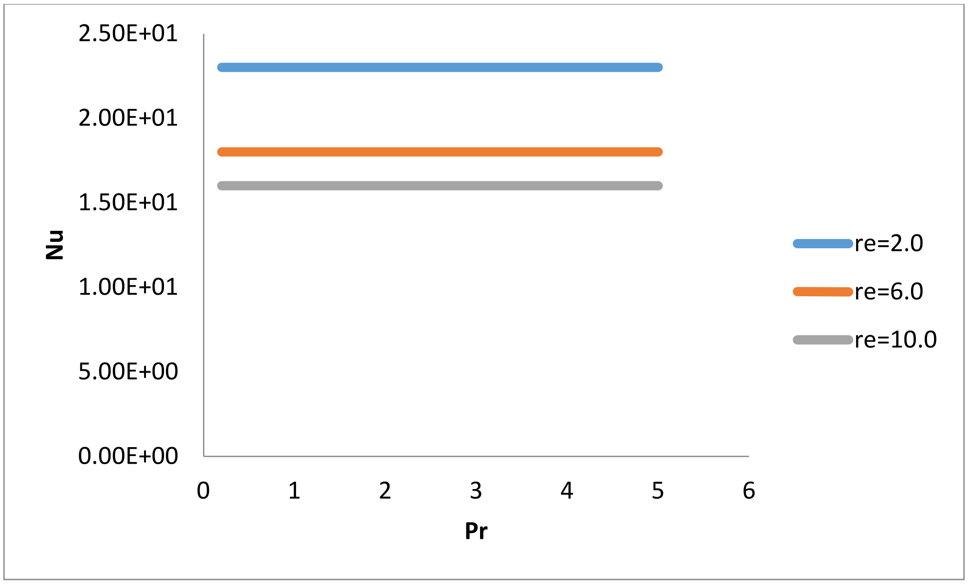

Fig 18: Nusselt number for various Re at y = 0

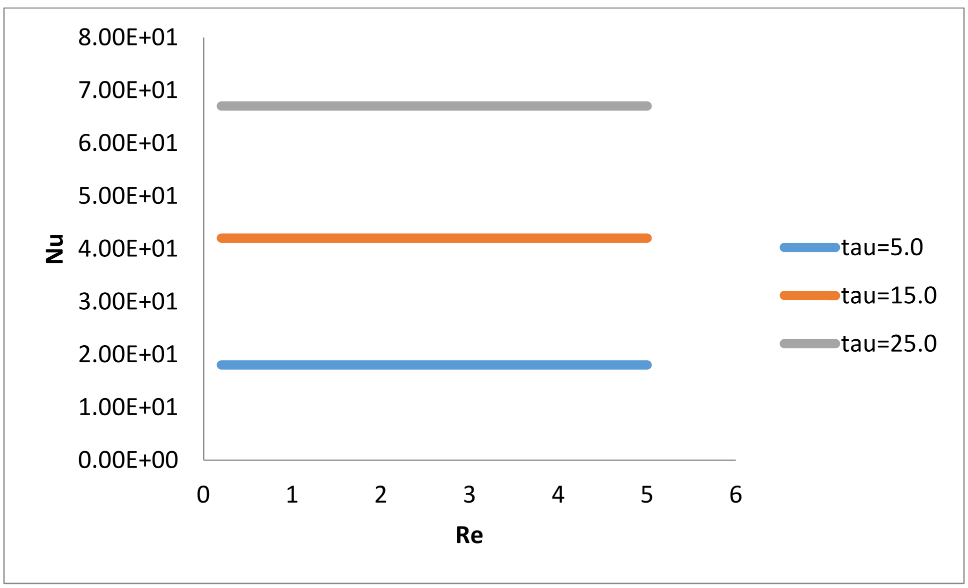

Fig. 19: Nusselt number for various τ at $y = 0$

Fig 17-19 analyzes the impact of Nusselt number at the lower plate. We can conclude that, the Nusselt number enhances with the enhancement of Reynolds number, mass concentration parameter and Prandtl number, decreases with the rise of Re and τ at the lower plate.

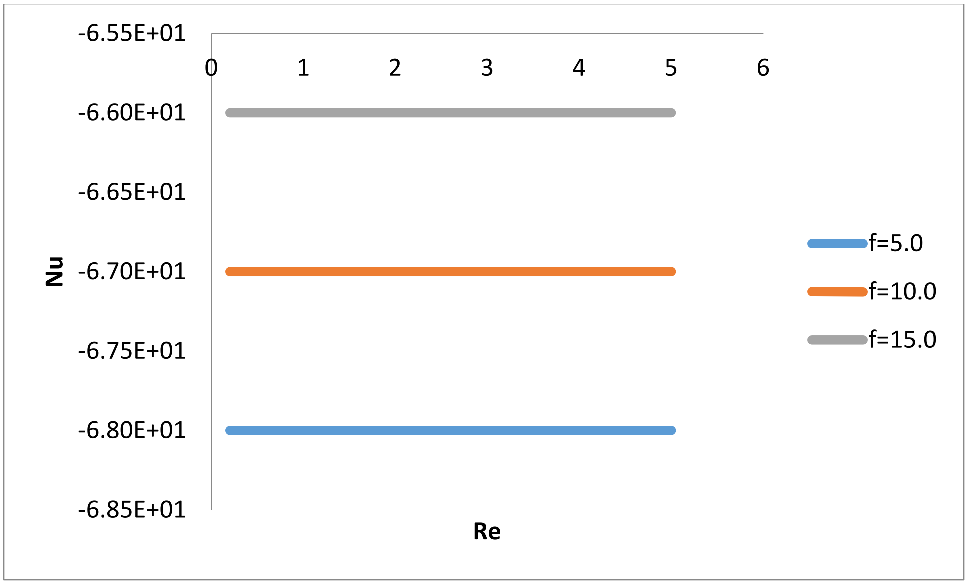

Fig 20. Nusselt number for various f at y = 1

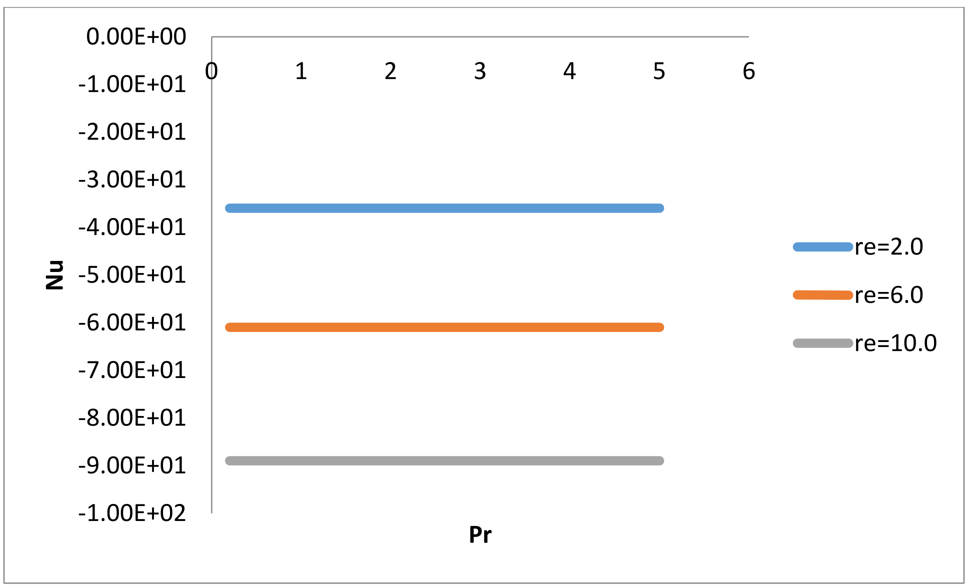

Fig 21. Nusselt number for various Re at y = 1

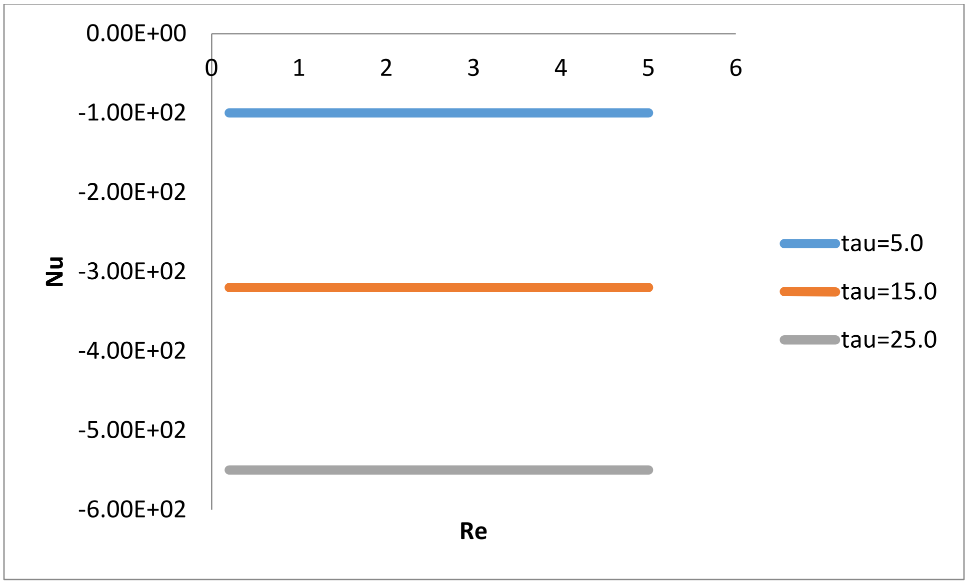

Fig 22. Nusselt number for various τ at y=1

Fig 20 – 22 depicts the effect of Nusselt number at the upper plate. It can be seen that, Nusselt number rises with the rise of Reynolds number and mass concentration parameter, depreciates with the rise of Re, Pr and τ.

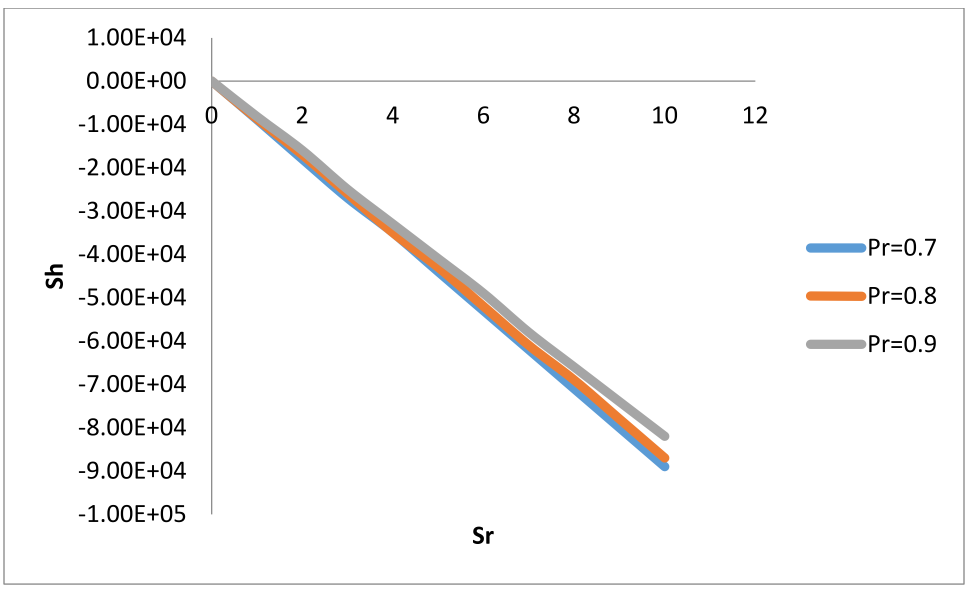

Fig 23: Sherwood number for various Pr (Air)

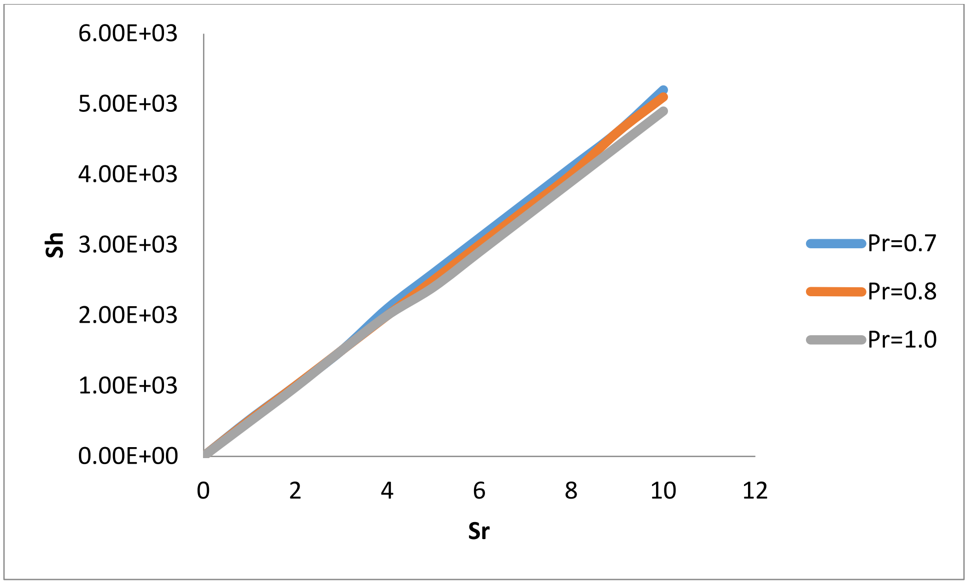

Fig 24: Sherwood number for various Pr (Air)

The impact of Sherwood number at lower and upper plates are portrayed in Fig 4.23 and 4.24. Form this figure, we can conclude that, Sherwood number increases at the lower plate and decreases at the upper plate with the increase in Soret number and Prandtl number.

5 Conclusion

The impact of pertinent parameters on velocity, temperature and concentration are analyzed over an incompressible viscous dusty fluid with thermo diffusion effect between the porous boundaries. The key findings are

- ✓ Main flow velocity at the fluid phase increases for enhancing mass concentration parameter and relaxation time parameter, depreciates with the enhancement of Reynolds number for various y.
- ✓ Main flow velocity at the particle phase increases with the increase in these parameters for various y.
- ✓ Concentration field is decreasing with increasing Prandtl number and enhances due to the increase of Soret number.
- ✓ Nusselt number enhances with the enhancement of Reynolds number, mass concentration parameter and Prandtl number, decreases with the rise of Re and τ at the both the plates.
- ✓ Sherwood number increases with the at the lower plate and decreases at the upper plate with the increase in Soret number and Prandtl number.

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