



Research Paper

"Axis-Based Analysis of the Dependent Pi Term Pt: A 2D Graphical Study Across OX, OY and OZ"

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ABSTRACT:

This study focuses on analyzing the dependent Pi term, Processing Time (Pt), using a 2D axis-based graphical approach. By plotting Pt against combinations of independent Pi terms along the X, Y, and Z axes, we aim to uncover the influence of each group of variables on processing time. The study applies dimensional analysis to traditional and modern wheat milling techniques, particularly household Chakki systems and roller mills. The graphical results indicate that Pt increases with the product of independent Pi terms, showing nonlinear trends and multiple peaks, suggesting that the process is governed by more than one mechanism. This work helps to better understand the performance of wheat grinding systems and optimize them for efficiency and quality.

Keywords: Dimensional Analysis, Pi Terms, Processing Time, 2D Graph, Chakki Milling, Roller Mill, Buckingham Pi Theorem, Axis-Based Plot, Wheat Grinding Process

INTRODUCTION:

The grinding of wheat into flour has evolved from traditional Chakki milling to modern roller mills, but the basic principles remain the same grains are broken and separated into bran, germ, and endosperm. In household atta chakkis, two circular stone disks grind the grain, with manual or mechanical control over the spacing between the disks. This simple system results in 15%– 17% starch damage and retains most of the bran and germ, making it nutritionally rich.

Figure: Atta Chakki of Yesteryears



Roller Mills

There are three sets each consisting of two rollers and associated sifters. The first set of rollers has corrugated surface, one moving faster than the other causing the brown skin (bran) of the wheat grain to be sheared away. The output is then sifted. After the bran has been removed, the output is passed through two more sets of rollers and sifters to separate germ and finally producing flour. Normally, this results in 5% to 6% damage to the starch and loss of ash.

In India, the corrugated surface of the first set of rollers is designed to actually crack the wheat grain and not just shear it. This technique results in comparable starch damage of 11% to 13% comparable to the Chakki milling and ash to 1% needed to make atta.

Literature Review:

Nataraja, Jayaram, and Ravikumar (2006) proposed a fuzzy-neuro model for normal concrete mix design. Their study demonstrated how intelligent systems can be effectively applied in civil engineering to improve accuracy in mix proportions. Similarly, Yatim et al. used Artificial Neural Networks (ANN) to evaluate building performance with a focus on service life planning, showing how ANN can contribute to sustainability in construction. Dawood and Al-Bazi (2008) developed an intelligent simulation model for managing manufacturing in the precast concrete industry, highlighting how computational tools can enhance planning and productivity.

In the field of concrete strength prediction, Alilou and Teshnehlab (2009) built an ANN model to predict 28-day compressive strength of concrete based on third-day data, proving that early prediction is possible with smart modeling. On the other hand, Dr. Michael O' Neill (2011) explored holistic ergonomics in the workplace, addressing the changing nature of work and emphasizing the need for human-centered design in evolving work environments.

Marras et al. (1995) identified key biomechanical risk factors that contribute to lower back disorders in occupational settings, underlining the importance of ergonomic assessment. Bapat and Modak (1994) created a generalized experimental model for a manually operated flywheel motor and explored its performance and optimization. Their work laid the foundation for sustainable energy applications using human power. Continuing this direction, Dhale and Modak (2012) formulated a data-based model for an oil press powered by a human-driven flywheel motor, supporting eco-friendly agricultural practices by replacing electrical energy with manual input.

Objectives of the Study:

1. To study the behavior of P_t by plotting it against combinations of independent P_i terms along the OX, OY, and OZ axes.
2. To observe patterns and trends in the 2D plots that show how P_t changes with the product of P_i terms.
3. To determine the number of peaks in the graphs and understand how many mechanisms are involved in the grinding process.

Hypothesis:

- H_0 (Null Hypothesis):
There is no significant relationship between the dependent P_i term (Processing Time, P_t or Electrical Energy Expenditure, E_e) and the independent P_i terms.

- H_1 (Alternative Hypothesis):

The dependent Pi terms (P_t and E_e) are significantly affected by the independent Pi terms, and the relationships are nonlinear, requiring multiple mechanisms to explain the observed variations.

Research Methodology:

This study applies dimensional analysis using the Buckingham Pi Theorem to model the wheat grinding process and the related energy consumption. The methodology includes the following steps:

1. Identification of Variables:

Physical variables involved in Chakki and roller milling (e.g., grain size, speed, time, power) were identified and converted into dimensionless Pi terms.

2. Formation of Pi Groups:

- Positive Pi terms: $\pi_2, \pi_3, \pi_4, \pi_6$
- Negative Pi terms: π_1, π_5, π_7

These were grouped into:

- π_a = Product of positive Pi terms
- π_b = Product of negative Pi terms
- π_8 (P_t or E_e) = Dependent Pi term (Processing Time or Energy Expenditure)

3. Graphical Analysis:

- 2D Graphs: The dependent Pi term was plotted along the Z-axis against combinations of positive (OX) and negative (OY) Pi terms on the X and Y axes.
- 3D Surface Graphs: Plots were created using combinations of π_a and π_b with π_8 .

4. Regression Modeling:

- Polynomial regression (Poly5) was applied using curve fitting tools to create mathematical models.
- Logarithmic transformation was used to simplify the exponential models, followed by matrix-based solution using MATLAB.

Analysis

1. 2D Plot Analysis of P_t

- Figure 7.6: 2D plot of dependent Pi term P_t across OX-OZ axes

A 5th-degree polynomial (Poly5) regression was fitted:

$$f(x) = p_1x^5 + p_2x^4 + p_3x^3 + p_4x^2 + p_5x + p_6$$

- Model Fit Summary:

- $R^2 = 0.7693$ (Reasonable fit)
- RMSE = 212.3
- Shows nonlinear trend and multiple peaks
- Conclusion: 8 peaks $\rightarrow$ 16 underlying mechanisms

- Figure 7.7: 2D plot of dependent Pi term P_t across OY-OZ axes

- Poly5 regression with lower $R^2 = 0.4104$
- Indicates moderate fit, supports multi-mechanism behavior

2. Model for Electrical Energy Expenditure (E_e)

- Based on Equation (7.14):

$$\pi_8(E_e) = 0.0578 \pi_1^{-0.994} \times \pi_2^{0.0307} \times \pi_3^{0.5022} \times \pi_4^{0.1313} \times \pi_5^{-0.0103} \times \pi_6^{0.0583} \times \pi_7^{-0.9144}$$

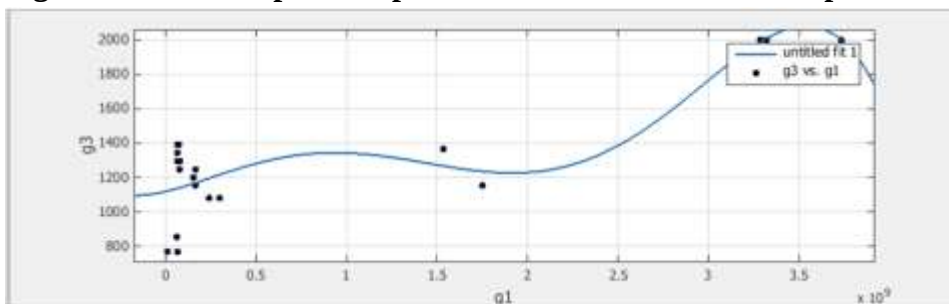
- Combined Pi terms:
 - $\pi a = \pi_2 \times \pi_3 \times \pi_4 \times \pi_6$
 - $\pi b = \pi_1 \times \pi_5 \times \pi_7$
- Transformed Model (Logarithmic):
Equation (7.21):
 $\log(\pi_8) = \log(a_0) + a_1 \log(\pi a) + a_2 \log(\pi b)$
- Matrix Method Used for Coefficient Calculation:
 - Solved using MATLAB
 - Coefficients obtained:
 - $a_0 = 17.374$
 - $a_1 = 0.1025$
 - $a_2 = -0.5508$
- Final Model:
 $\pi_8(Ee) = 17.374 \times \pi a^{0.1025} \times \pi b^{-0.5508}$ (Equation 7.24)

3. Graphical Analysis for Ee

- Figure 7.8: 2D Plot of all independent Pi terms vs. dependent Pi term (Ee)
 - Shows nonlinear growth of Ee with increase in product of Pi terms
 - 8 peaks observed $\rightarrow$ 16 mechanisms involved
- Conclusion: Both Pt and Ee exhibit nonlinear behavior and cannot be modeled by a single mechanism. Graphical and regression analysis confirms the multi-mechanistic nature of the milling process.

Analysis of the study:

Figure 7.6 :2-D Graph of all points on the OX OZ axis for dependent Pi term Pt:



Linear model Poly5 is formulated using curve fitting tool to get the regression equation using least square method.

Linear model Poly5:

$$f(x) = p1 * x5 + p2 * x4 + p3 * x3 + p4 * x2 + p5 * x + p6$$

(7.12) Coefficients (with 95% confidence bounds):

$$p1 = -4.491e - 44 (-3.28e - 43, 2.381e - 43)$$

$$p2 = 3.49e - 34 (-2.187e - 33, 2.885e - 33)$$

$$p3 = -8.067e - 25 (-8.377e - 24, 6.763e - 24)$$

$$p4 = 4.821e - 16 (-7.912e - 15, 8.876e - 15)$$

$$p5 = 2.444e - 07 (-2.812e - 06, 3.301e - 06)$$

$$p6 = 1119 (855.4, 1383)$$

Goodness of fit:

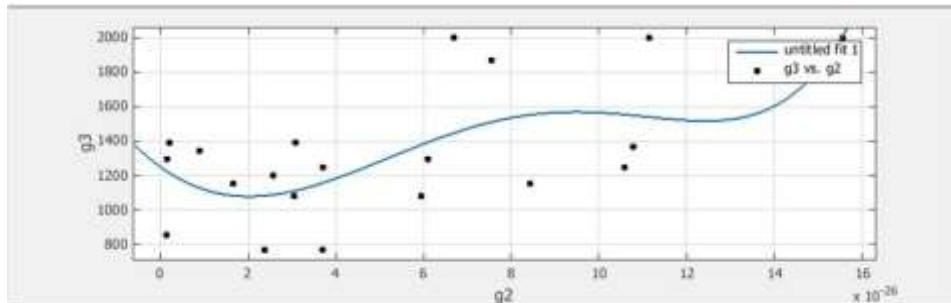
SSE: 6.309e+05

R-square: 0.7693

Adjusted R-square: 0.6869

RMSE: 212.3

Figure 7.7: 2-D Graph of all points on the OY OZ axis for dependent π_i term Pt:



Linear model Poly5 is formulated using curve fitting tool to get the regression equation using least square method.

Linear model Poly5:

$$f(x) = p1 * x^5 + p2 * x^4 + p3 * x^3 + p4 * x^2 + p5 * x + p6$$

(7.13) Coefficients (with 95% confidence bounds):

$$p1 = 1.108e + 128 (-2.598e + 129, 2.82e + 129)$$

$$p2 = -1.378e + 103 (-9.921e + 104, 9.645e + 104)$$

$$p3 = -3.181e + 78 (-1.251e + 80, 1.188e + 80)$$

$$p4 = 5.61e + 53 (-5.643e + 54, 6.765e + 54)$$

$$p5 = -1.844e + 28 (-1.332e + 29, 9.627e + 28)$$

$$p6 = 1250 (726.8, 1773)$$

Goodness of fit:

SSE: 1.613e+06

R-square: 0.4104

Adjusted R-square: 0.1998

RMSE: 339.4

Formulation of Models Based on combination of observed data for dependent π_i term Electrical Energy Expenditure

Model for dependent variable: Electrical Energy Consumption

$$\pi_8(E_e) = 0.0578 \pi_1^{-0.994} * \pi_2^{0.0307} * \pi_3^{0.5022} * \pi_4^{0.1313} * \pi_5^{-0.0103} * \pi_6^{0.0583} * \pi_7^{-0.9144}$$

(7.14)

Two more independent π_i terms (π_a, π_b) were formed and already formed one dependent π_i term (π_8) were decided during experimentation and hence are available for the model formulation.

π_a is formed by the product of the positive independent π as specified in equation (7.14).

$$\pi_a = (\pi_2 * \pi_3 * \pi_4 * \pi_6)$$

(7.15)

π_b is formed by the product of the negative independent π as specified in equation (7.14).

$$\pi_b = (\pi_1 * \pi_5 * \pi_7)$$

(7.16)

Each dependent π term is the function of the available independent terms,

$$\pi_8(Ee) = f(\pi_a, \pi_b)$$

(7.17)

A probable exact mathematical form for the dimensional equations of the phenomenon could be relationships assumed to be of exponential form ^[26]. For example, the model representing the behavior of dependent π term π_8 with respect to various independent π terms can be obtained as under.

$$\pi_8(Ee) = a_0 \pi_a^{a_1} * \pi_b^{a_2}$$

(7.18)

Therefore two unknown terms in the equation 10 i.e. constant of proportionality a_0 & indices a_1, a_2 .

The values of exponent are a_1 and a_2 are established independently at a time, on the basis of data collected through classical experimentation. There are three unknown terms in the equation (7.18) curve fitting constant a_0 and indices a_1 and a_2 . To get the values of these unknowns we need minimum a set of three set of all unknown dimensionless π terms.

$$Z = A + b_X + C_Y + \dots$$

(7.19)

The equation (7.18) can be brought in the form of equation (7.19) by taking log on both sides.

Model of dependent π term π_8 for process time

$$\pi_8(Ee) = a_0 \pi_a^{a_1} * \pi_b^{a_2}$$

Taking log on the both sides of equation for π_8

$$\text{LOG} \pi_8 = \text{LOG} a_0 + a_1 \text{LOG} \pi_a + a_2 \text{LOG} \pi_b$$

(7.20)

Let, $Z_1 = \log \pi_8$, $K_1 = \log a_0$, $A_1 = \log \pi_a$, $B_1 = \log \pi_b$

Putting the values in equations 4, the same can be written as

$$Z_1 = K_1 + a_1 * A_1 + a_2 * B_1$$

(7.21)

Equation (7.21) is a regression equation of Z on A and B in a dimensional coordinate system

$$\sum Z_1 = n * K_1 + a_1 \sum A_1 + a_2 \sum B_1$$

(7.22)

$$\sum Z_1 A_1 = K_1 \sum A_1 + a_1 \sum A_1 * A_1 + a_2 \sum B_1 * A_1$$

$$\sum Z_1 B_1 = K_1 \sum B_1 + a_1 \sum A_1 * B_1 + a_2 \sum B_1 * B_1$$

(7.23)

In the above set of equations, the values of the multipliers A_1 , B_1 are substituted to compute the a_0 , a_1 and a_2 in the set of equations.

After substituting these values in the equations (7.22) one will get a set of seven equations, which are mutually to get the values of a_0 , a_1 and a_2 . The above equations can be verified in the matrix form and further values of a_0 , a_1 and a_2 can be obtained by using matrix analysis.

$$X1 = \text{inv}(W) * P1$$

Solving these equations using 'MATLAB' is given below. $W = 3 \times 3$ matrix multipliers of a_0 , a_1 and a_2

$P_1 = 3 \times 1$ matrix of the terms on L H S and $X_1 = 3 \times 1$ matrix of values of a_0 , a_1 and a_2

For deduction of Combined independent Pi term π_a and π_b for Response variable Electrical Energy Expenditure Time from Observation Table refer Annexure 40

For Deduction of Combined independent Log Pi term π_a and π_b for Response variable Electrical Energy Expenditure from Observation Table refer Annexure 41

For calculation of matrix formation using Log Values of independent and dependent Pi values (Electrical Energy Expenditure) refer Annexure 42

Matrix Transformation and deduction of indices for model Formulation for Electrical Energy Expenditure is depicted in Annexure 43

Solving Matrix Transformation and deduction of indices for model Formulation for Electrical Energy Expenditure using MATLAB is depicted in Annexure 44

After solving we get

$$a_0 = 17.374, a_1 = 0.1025, a_2 = -0.5508$$

Hence the model is

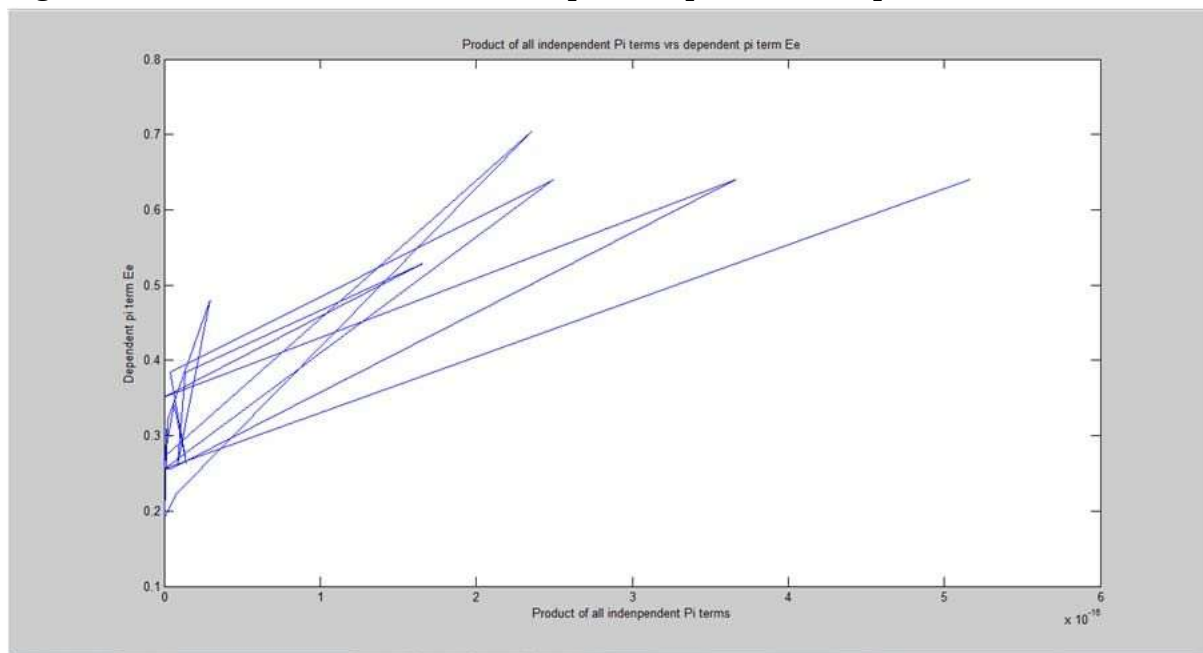
$$\pi_8(Ee) = 17.374 \pi_a^{0.1025} * \pi_b^{-0.5508}$$

(7.24)

7.1.1.1 Graphical Analysis of combination of observed data for Individual Mathematical Model for dependent pi term Electrical Energy Expenditure (Ee) 2-D and 3-D graphs are plotted for all the dependent pi terms.

To obtain 2-D graph, dependent pi term is plotted on Z axis, whereas the product of positive pi terms (π_2 , π_3 , π_4 and π_6) and negative pi terms (π_1 , π_5 and π_7) is plotted on the x and y axis respectively.

To obtain 3-D surface graph, dependent pi term is plotted on Z axis, whereas the product of positive pi terms (π_2 , π_3 , π_4 and π_6) and negative pi terms (π_1 , π_5 and π_7) is plotted on the x and y axis respectively.

Figure 7.8: 2-D Plot of Product of all independent pi terms vs dependent Pi term Ee:

It can be observed from the plot that as the product of the independent pi terms increases, the Electrical Energy Expenditure time Ee, also tends to gradually increase. It is evident from the graph that this phenomenon cannot be represented by a single mechanism. Presence of 8 peaks in the graph makes needs involvement of 16 mechanisms imperative.

Conclusions Overall Results:

1. The study successfully used dimensional analysis and the Buckingham Pi theorem to understand complex interactions in the wheat grinding process.
2. Through 2D and 3D graphical analysis, it was observed that the dependent Pi terms Pt and Ee increase as the product of independent Pi terms increases, but not in a linear way.
3. Multiple peaks in the graphs (7– 8 peaks) indicate that multiple mechanisms (14– 16) are involved in the grinding and energy expenditure processes.
4. Polynomial regression (Poly5) and logarithmic models were formulated to represent these interactions mathematically. For instance, the final model for energy expenditure was:

$$\pi_8(Ee) = 17.374 \times \pi_a^{0.1025} \times \pi_b^{-0.5508}$$
5. The goodness of fit (R^2) for most models was moderate to strong, showing that the models are reliable in predicting behavior within the observed range.
6. This approach confirms that dimensional modeling and axis-based plotting are effective tools to study traditional systems like atta Chakki and modern roller mills.

Future Scope of the study:

- **Experimental Validation:** Future work can include real-time data collection from both traditional Chakki mills and industrial roller mills to validate the Pi models.
- **Advanced Modeling:** Machine learning tools such as Artificial Neural Networks (ANN) can be used alongside Pi models to improve prediction accuracy.
- **Automation and Control:** These models can help in designing automatic control systems to optimize processing time and reduce energy use.
- **Application to Other Systems:** The same dimensional modeling approach can be applied to other food processing systems (like rice milling, oil extraction, etc.).

- Ergonomic and Environmental Studies: Future research can explore the health, ergonomic, and environmental impacts of prolonged wheat grinding using these models.
- Energy Optimization: Further analysis can focus on minimizing electrical energy expenditure while maintaining product quality.

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